Single Carrier Digital Terrestrial Television Broadcasting

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***Abstract*—A transmission system for digital terrestrial television broadcasting has been designed. This system is based on the European cable system but uses stronger error correction and better equalization. The stronger error correction is a concatenation of Reed Solomon coding RS[204,188,17] and convolutional coding with R_{conv} =1/2, 2/3, 3/4, 5/6 and 7/8. The algorithm which is used for convolutional decoding is the Viterbi algorithm. To provide the Viterbi decoder with soft decision information, every symbol bit will be expanded with two soft decision (reliability) bits. The modulation scheme of the terrestrial transmission system is 64-QAM square root raised cosine filtered with a roll off factor $\alpha = 0.15$. The mapping of the symbols into the 64-QAM constellation is a Gray-mapping over the complete I, Q-plane. In this paper the performances of the terrestrial transmission system are simulated and analyzed.**

***Keywords*—Digital terrestrial television broadcasting, channel coding, reliability estimation, quadrature amplitude modulation, digital video.**

I. Introduction

Digital transmission through a terrestrial channel is important for digital TV services. Therefore, PHILIPS RESEARCH has developed and analyzed a digital transmission scheme that is appropriate for broadcasting digital TV services through a terrestrial channel. The SC-DTTB (Single Carrier Digital Terrestrial Television Broadcasting) system is able to transport an MPEG-2 (Moving Pictures Experts Group-2) transport stream at a single carrier in the presently used UHF-band (Ultra High Frequency-band) or at a higher frequency-band e.g. the 2.5 GHz band. The SC-DTTB system is based on the standard system for satellite and cable described in respectively [1], [2]. The transmitter power by satellite transmission is low and the TWTA (Traveling Wave Tube Amplifier) is highly nonlinear. The transponder bandwidth is 26-54 MHz and the transmission medium approaches the AWGN (Additive White Gaussian Noise) channel. These characteristics of the transmission system have led to the choice of QPSK (Quaternary Phase Shift Keying) modulation with a FEC (Forward Error Correcting) code consisting of Reed-Solomon outer coding and convolutional inner coding. The cable channel is bandwidth limited at 8 MHz with a high signal to noise ratio. The cable channel is a moderate channel because the only distortions which appear are short term echoes due to impedance mismatches at the terminal outputs. The small channel bandwidth in combination with the high signal to noise ratio and the weak distortions has led to choose 64- QAM (Quadrature Amplitude Modulation) with only Reed-Solomon error correction. The terrestrial channel is band-limited at 8 MHz and is described by highly dispersive multi-path caused by reflection and scattering. The paths between transmitter and receiver may be seen to consist of large reflectors and/or scatter-ers at some distance from the receiver, giving rise to a number of waves that arrive in the vicinity of the receiver with random amplitudes and delays. The multi-path propagation generates frequency selective channel transfer characteristics and produces ISI (Inter-Symbol Interference) in the digital signal. The terrestrial system behavior lies between the cable system and satellite system. This means it is worse than the cable system but better than the satellite system. Thus it should be possible to compose the SC-DTTB system with components of the satellite system and the cable system. The band-limitation in combination with a large as possible coverage area (limited by the signal strength) has led to the choice of 64-QAM (cable modulation scheme) with a concatenated FEC code of Reed-Solomon outer coding and convolutional inner coding (satellite error correcting scheme) for the SC-DTTB system. Fig.1 shows the block-diagram of the SC-DTTB system.

II. General description of the SC-DTTB system

The major differences between the cable and satellite system is the forward error correction and the modulation method. The cable system uses 64-QAM modulation while the satellite system uses QPSK-modulation. The

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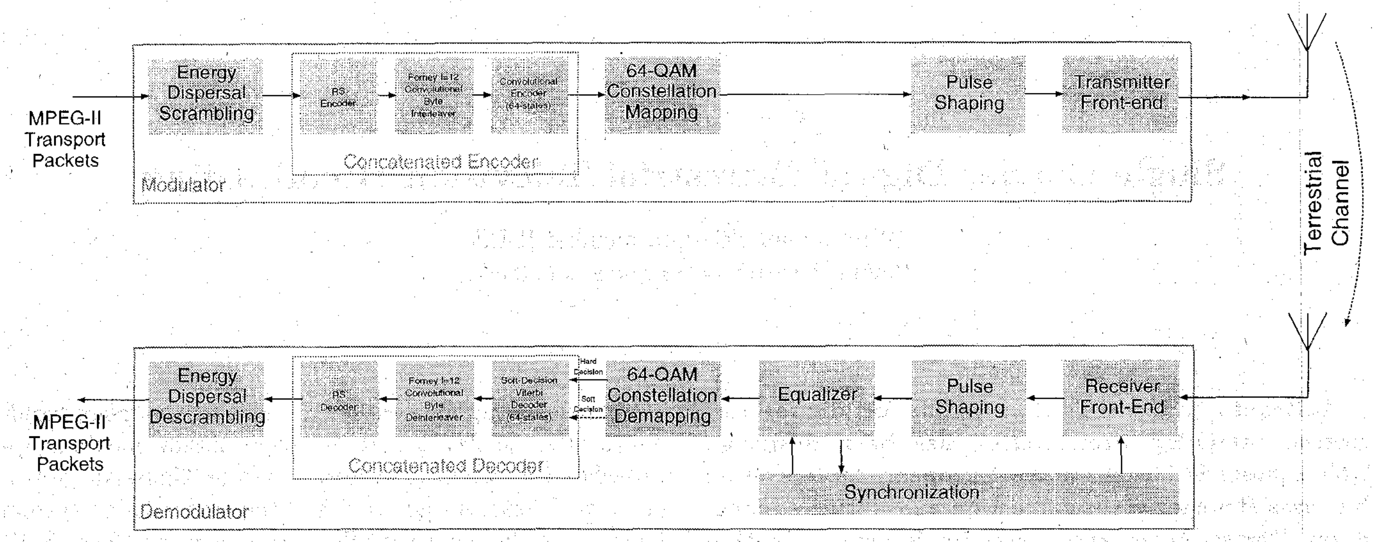

Fig. 1. The block-diagram of the SC-DTTB system.

FEC of the satellite system consists of concatenated coding with Reed-Solomon outer coding and convolutional inner coding while the cable system only uses Reed-Solomon coding. Since the SC-DTTB system is a combination of the cable system and the satellite system it is not necessary to discuss the SC-DTTB system in detail. Because this can be found in [1], [2], so the common system blocks will be described in a more general way and the specific SC-DTTB blocks in detail.

A. Modulator

1) Energy Dispersal Scrambling: In order to comply with radio regulations and to ensure adequate binary transitions, the MPEG-2 data will be scrambled. The scrambling method is based on a so called, "pseudo random bit sequence." This sequence has a flat power spectrum, so the energy will be equally dispersed over the whole spectrum. The pseudo random sequence of pulses is generated by a LFSR (shift register with linear feedback). The polynomial of the LFSR is given by

$$1 + X^{14} + X^{15}. \tag{1}$$

To ensure synchronization of the de-scrambler only the data will be scrambled. The LFSR is only enabled during the actual MPEG-2 data and disabled during synchronization bytes. To provide an initialization of the de-scrambler every sync byte of the first transport packet in a group of eight packets is bitwise inverted [2].

2) RS Encoder: Reed-Solomon codes are non-binary linear block-codes [3]. A linear block code consists of a set of fixed length code words in which the element of the code words are selected from an alphabet of q symbols. Usually $q = 2^p$ so that p information bits are mapped onto a symbol. A codeword has length n, dimension k and $n-k$ redundant symbols.

The minimum Hamming distance between two code words is denoted as d_{min}. The number of errors that can be corrected by the Reed-Solomon codes are denoted by the letter t and given by

$$t = \frac{d_{min} - 1}{2}. \tag{2}$$

Hence, a code can be characterized by three parameters; the block length n, symbols per block k and hamming-distance d. This three parameters are normally given as $RS[n, k, d]$ to indicate the type of Reed-Solomon code. Since Reed-Solomon codes are maximum distance separable (MDS) [3]

$$d_{min} = n - k + 1. \tag{3}$$

The $RS[204, 188, t = 8]$, shortened code from the original $RS[255, 239, t = 8]$ code, is in this system applied to each randomized transport packet of $k = 188$ bytes. Substituting the parameters $n = 204$ and $k = 188$ in (3), this yields, $d_{min} = 17$. Substituting this value in (2) gives $t = 8$, the number of bytes that can be corrected. The code can also be presented by $RS[204, 188, 17]$ which is a more common notation. The redundancy $(n - k) = 2t$, which is needed to achieve an error protection of correcting up to t errors, will yield a code rate

$$R_{RS} = \frac{k}{n} = \frac{188}{204}. \tag{4}$$

3) Forney $I = 12$ Convolutional Byte Interleaver: The Forney $I = 12$ convolutional byte interleaver is used to interleave the error protected packets to improve the performance of the system. The interleaving depth $I = 12$ is the total number of branches of the interleaver. Each branch, except the first one, contains a FIFO (First In First Out shift-register). The total number n of bytes per error protected packet has to be divided over the

I branches because the first byte (synchronization byte) of each MPEG-2 packet will be routed through branch $j = 0$. This is needed to provide a good synchronization of the MPEG-2 data stream. This division determined the length of the FIFO per branch and thus the value of M

$$M = \frac{n}{I}. \tag{5}$$

For the SC-DTTB system with $n = 204$ bytes and $I = 12$ becomes $M = 17$. The length of the FIFO per branch j is given by

$$D_j = M \cdot j \quad j = 0, 1, \ldots, I - 1. \tag{6}$$

4) Convolutional Encoder (64-stages): The principle of the convolutional encoder is shown by Fig.2, [4]. From

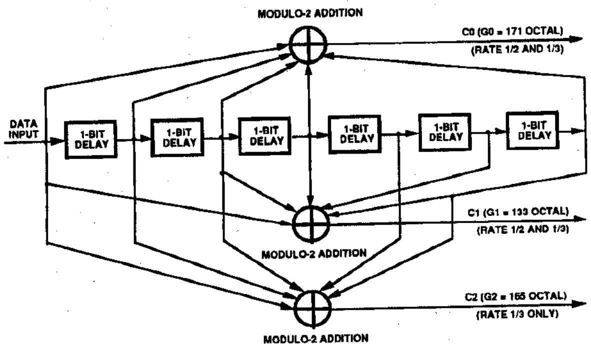

Fig. 2. The principle of the convolutional encoder

Fig.2 it is seen that the output of the encoder is determined by six previous bits plus the current input bit. This is a constraint length 7 code denoted as $K = 7$, [4]. From Fig.2 it is also seen that there is one input bit and there are two or three output bits, so coding rates

$$R = R_{conv} = \frac{\text{input bits}}{\text{output bits}} \tag{7}$$

of $R = \frac{1}{2}$ and $R = \frac{1}{3}$ are possible (code rate $R = \frac{1}{3}$ is not used in the SC-DTTB system). In addition to these fundamental code rates other higher code rates are achievable through a punctured coding technique.

Punctured coding techniques allow a lower data rate to be used on the communication channel than within the encoder/decoder. This is possible because some bits of the data rate are punctured or deleted in a repeating pattern and they are not transmitted. Fig.3 shows the principle for $R = 1/2$ to $R = 3/4$, [4].

5) 64-QAM Constellation Mapping: This block performs exact mapping of bits onto QAM symbols. This means that the first incoming bit is mapped on the MSB (Most Significant Bit) of symbol j. This method of direct bit mapping proceeds until the least significant bit (LSB) of the symbol j has been defined. Then the following incoming bit will be mapped as the MSB of symbol $j+1$ and

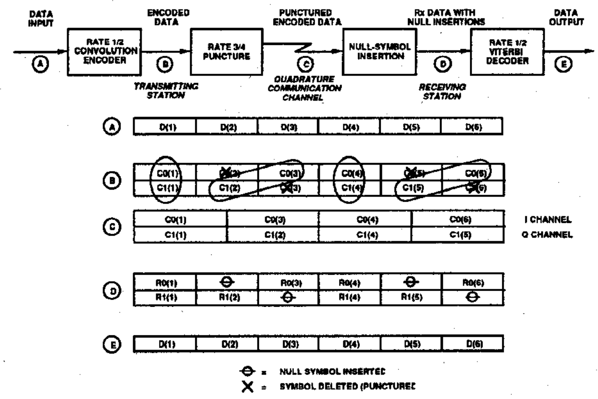

Fig. 3. The principle of puncturing

this mapping proceeds further on downwards. If the bits are mapped on the QAM symbols they are represented by a QAM constellation diagram. A 64-QAM constellation diagram is composed in such a way that it can be represented by two times 8-ASK (eight level Amplitude Shift Keying) modulation with Gray-mapping. This means that every symbol in the complex I,Q-plane only differs one bit from its adjacent symbols this is shown in Fig.4 with d

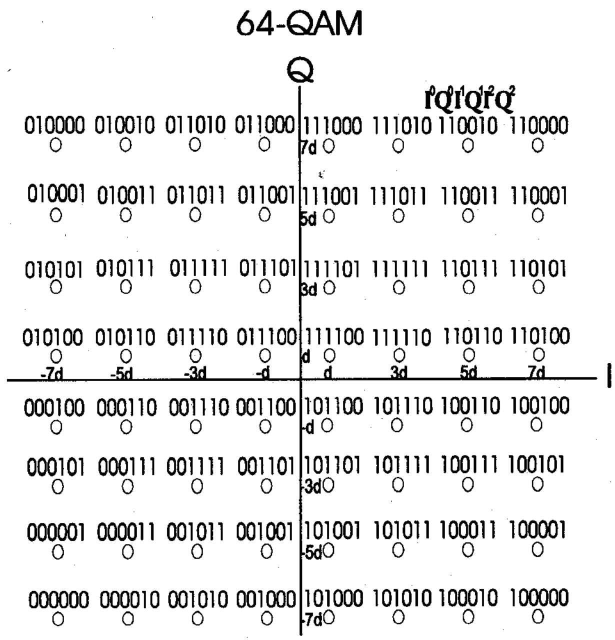

Fig. 4. 64-QAM constellation diagram with Gray-mapping

is the distance between the symbol and its decision boundary. The practical problem of the 2^m-QAM (with m the bits per symbol) constellation is that it is rotationally invariant for some particular angles of rotation, typically multiples of $\pi/2$. If the constellation is rotated, there is no way that the receiver can distinguish it from a valid constellation, unless it knows the actual rotation. This problem can be eliminated by using feedback information of the Viterbi decoder. In the case of a rotated constellation diagram (almost) all received symbols are incorrect and now, to obtain correct symbols, switching over to another quadrant is required. This switching over to another quadrant is based on information out of the Viterbi decoder.

6) Pulse Shaping: The I and Q signal will be square root raised cosine filtered to minimize ISI. The roll off factor is 15% also denoted as $\alpha = 0.15$. The roll off factor is an important parameter to calculate the relative transmission rate ζ, defined as the ratio of the symbol rate R_s and the according channel bandwidth B (normally the maximum available bandwidth),

$$\zeta \equiv \frac{R_s}{B} = \frac{R_s}{R_s \cdot (1+\alpha)} = \frac{1}{(1+\alpha)}, \; \alpha \geq 0. \tag{8}$$

The unit of the relative symbol transmission rate ζ is symbols/s/Hz. Actually is ζ a scale factor for the bandwidth efficiency due to Nyquist pulse shaping. Hence, with a maximum channel bandwidth $B = 8$ MHz for the SC-DTTB system the useful bandwidth W becomes

$$W = \zeta \cdot B = \frac{1}{(1+\alpha)} \cdot B = 0.87 \cdot 8 = 6.96\, MHz. \tag{9}$$

7) Transmitter Front-end: This block performs up-conversion of the QAM-signal to the appropriate channel frequency for terrestrial broadcasting.

B. Demodulator

In this section only the demodulator operations are described which have no counterpart at the modulator side or are different from the operation performed at the modulator side.

1) Equalizer: Due to the introduction of echoes by multi-path and by imperfect filter realization there will be ISI. The transmission scheme 64-QAM is rather sensitive to ISI because the distance between adjacent symbols in the constellation diagram is small with respect to the length of the vector which represents the symbols at the corners of the constellation diagram. This ISI problem can be solved by an equalizer which tries to eliminate the ISI introduced by the channel. To obtain some feeling how sensitive 64-QAM is for echoes and the necessity for equalization, a simulation with the one ray beam multi-path channel model is performed. This model provides a direct signal path and a single multi-path ray beam that is time delayed with τ, phase shifted with $2\pi f_o \tau$ and attenuated with β. This model is represented by

$$s_{out}(t) = s_i(t) - \beta e^{j2\pi f_o \tau} s_i(t-\tau) \tag{10}$$

The frequency response of the one ray beam multi-path channel $H_b(f)$ is achieved by Fourier-transformation of (10)

$$H_b(f) = \frac{S_{out}(f)}{S_i(f)} = 1 - \beta e^{-j2\pi(f-f_o)\tau}. \tag{11}$$

The ISI which is caused by the one ray beam multi-path channel with an echo at $\tau = 1\mu$s, an amplitude gain of $\beta = 0.1$ (-20 dB) and $f_0 = 0$ kHz almost closes the eye of 64-QAM signal and the vector points in the constellation diagram are widely scattered.

The degradation of the eye will degrade the SER (Symbol Error Ratio) performances because the average distance between a symbol and its decision boundary is decreased. If the signal is contaminated with AWGN, less noise is needed to make a wrong decision compared with the case when there is no ISI. This degradation in performances is shown by Fig.5 where the SER (only the exponent of the power of ten is shown) is given as function of the E_b/N_0 (is the ratio between the energy per useful bit and twice the noise spectral density) for 64-QAM signal. With one echo at $\tau = 1\mu$s and with different amplitude gains of $\beta = 0.1\ (-20dB), 0.05\ (-26dB), 0.01\ (-40dB)$ and $f_0 = 0$ kHz. From Fig.5 can be seen that the SC-DTTB system needs a good equalizer to cope with this kind of ISI distortion. The SC-DTTB system uses a equalizer which has the structure of a transversal filter with a TDL (tapped delay line) with adjustable weights $\{w_i\}$ and with an LMS (Least Mean Square) update algorithm. The adjustable weights (filter coefficients) of the equalizer determine the transfer function of the TDL. The adaptation of the equalizer to the channel is being accomplished by changing the weights $\{w_i\}$.

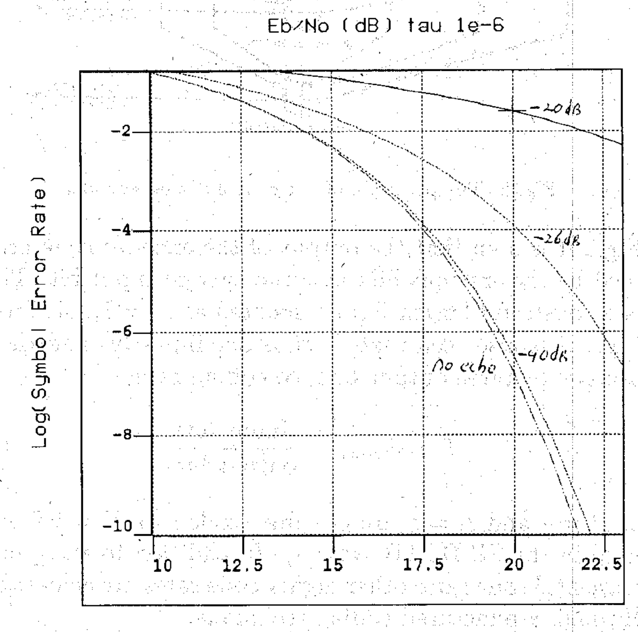

Fig. 5. Performances degradation due to echoes at $\tau = 1\mu s$ with amplitude gains of $\beta = 0.1\ (-20dB), 0.05\ (-26dB), 0.01\ (-40dB)$ and $f_0 = 0$ kHz.

2) 64-QAM Constellation Demapping: To provide the Viterbi decoder with reliability information, the received signal has to be examined at its reliability. If a received signal lies very close to a decision boundary, an error can occur easily with a little increase of the noise. Hence, the received signal near a decision boundary has a high level of unreliability. The level of reliability has to be fed as soft decision information to the Viterbi decoder.

The next question is how to estimate/determine the reliability of the received signal? A widely accepted criterion to determine reliability is the a posteriori probability $P[m_i \mid \underline{r} = \underline{\rho}]$ with the used parameters showed in Fig.6 [5]. The a posteriori probability $P[m_i \mid \underline{r} = \underline{\rho}]$ means the probability on the signal estimation $\hat{m}$ if vector $\underline{\rho}$ is received [5]. If the received signal is reliable the a posteriori probability $P[m_i \mid \underline{r} = \underline{\rho}]$ nearly equals one and also the smaller $P[m_i \mid \underline{r} = \underline{\rho}]$ the smaller the reliability of $\hat{m}$. The optimum receiver set $\hat{m} = m_k$ if and only if

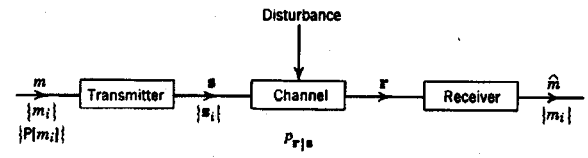

Fig. 6. a communication system with its parameters.

$$P[m_k \mid \underline{r} = \underline{\rho}] > P[m_i \mid \underline{r} = \underline{\rho}]$$
$$i = 0, 1, ..., L-1 \;\; i \neq k \tag{12}$$

with L the number of messages at the transmitter. This means the maximum a posteriori probability is detected for symbol m_k, such a receiver is called a MAP (Maximum A posteriori Probability) receiver. This receiver determines $\hat{m}$ from the knowledge of $P[m_i \mid \underline{r} = \underline{\rho}]$ or $P[\underline{s}_i \mid \underline{r} = \underline{\rho}]$ for a received vector $\underline{r} = \underline{\rho}$ or every point $\underline{\rho}$ in the complex plane. Hence, the complex plane is partitioned into disjoint regions I_i. Each region comprises all points such that whenever the received vector $\underline{r}$ is in I_k the optimum receiver sets $\hat{m}$ equal to m_k. The regions I_i are called "optimum decision regions", so the optimum receiver makes an error when $m = m_k$ if and only if $\underline{r}$ falls outside I_k. The boundaries of the decision regions depend on the a priori probability $P[m = m_i]$, and the definition of the channel. These boundaries are derived in App. A. For the 64-QAM constellation these boundaries are half the Euclidean distance between two adjacent points if every signal has equal probabilities and transmitted through an AWGN channel. The knowledge about the place of the boundaries is needed to obtain the reliability or soft decision information for the Viterbi decoder. The use of two-level quantization at the input of the Viterbi decoder is commonly referred as "hard decision" decoding. When the number of quantization levels is larger than two, the decoding is called "soft decision" decoding. The Viterbi decoder discussed in this paper processes symbol bits which are converted into 3 bits (eight-level) soft-decision information. The main advantage of eight-level soft-decision quantization over two-level hard-decision quantization is that it provides the decoder with more information in the form of a single polarity ("hard-decision") bit and two additional reliability bits. The additional reliability bits provide the decoder with a measure of the probability that the code symbol was correctly demodulated. Normally, the 3-bit soft decision information can be fed into the Viterbi decoder in either an offset-binary or sign-magnitude format. Fig.7 shows the offset binary and sign-magnitude data input encoding formats for soft-decision decoding [4]. To

	Offset-Binary Format			Sign-Magnitude Format		
R0[x], R1[x], or R2[x] Bit:	[2]	[1]	[0]	[2]	[1]	[0]
Strongest 1:	1	1	1	1	1	1
	1	1	0	1	1	0
	1	0	1	1	0	1
Weakest 1:	1	0	0	1	0	0
Weakest 0:	0	1	1	0	0	0
	0	1	0	0	0	1
	0	0	1	0	1	0
Strongest 0:	0	0	0	0	1	1

Fig. 7. Data input encoding formats for soft-decision decoding

convert the I and Q values into the appropriate bits for the Viterbi decoder the 64-QAM modulation scheme will be seen as two times 8-ASK modulation (with amplitude distance of $2d$) for the signals I and Q. This is permitted because the SC-DTTB system constellation diagram is a multiplexed composition of the same Gray coded 8-ASK signals for the in-phase (I) components and the quadrature (Q) components. The format of the multiplex composed symbols is shown in the first quadrant of Fig.4. This special mapping results in an independency of the I and Q components regarding their decision boundaries and results also in a Gray coded constellation diagram over the complete I, Q-plane. Every 3 bits (8-ASK) symbol will be converted into 3 times 3 bits because every symbol bit will be expand with two soft decision bits. Assume that the received value of the I signal is r_x and of the Q signal is r_y. The signals r_x and r_y are processed the same. Therefore only the operation on the r_x signal will be discussed. The Viterbi decoder needs 3 bits soft decision information for each of the 3 information bits represented by r_x. These will be obtained from de-mapping the value r_x and add two bits soft decision information to the hard decision symbol value. One symbol is represented by the signal r_x and this signal is converted into nine bits $r_0[0], r_0[1], r_0[2], r_1[0], r_1[1], r_1[2], r_2[0], r_2[1]$ and $r_2[2]$. The values of these bits are dependent of the Gray mapping of the constellation points. Fig.8 shows an 8-ASK constellation diagram with a Gray mapping which results into the 64-QAM constellation diagram of Fig.4.

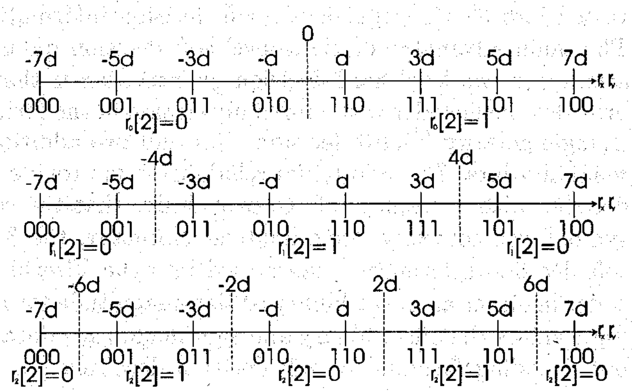

Fig. 8. 8-ASK decision regions with Gray mapping.

From Fig.7 is seen that $r_i[2]$ with $i = 0, 1, 2$ can also be interpreted as the hard decision bit because it is the sign bit of the soft decision information. Using the Gray mapping represented by Fig.8 the hard decision bits ($r_i[2]$, $i = 0, 1, 2$) are

$$r_0[2] = \frac{1}{2} \cdot [1 + \mathrm{sgn}(r_x)] \quad (13)$$

$$r_1[2] = \frac{1}{2} \cdot [1 + \mathrm{sgn}\,(4d - |r_x|)] \quad (14)$$

$$r_2[2] = \frac{1}{2} \cdot [1 + \mathrm{sgn}\,(2d - |4d - |r_x||)] \quad (15)$$

where $7d$ is the maximum amplitude of r_x and

$$\mathrm{sgn}(\chi) = \begin{cases} -1 & if\ \chi < 0 \\ 1 & if\ \chi \geq 0. \end{cases} \quad (16)$$

The soft decision information is then calculated by

$$\epsilon_0 = \frac{|r_x|}{7d} \quad (17)$$

$$\epsilon_1 = \frac{|4d - |r_x||}{4d} \quad (18)$$

$$\epsilon_2 = \frac{|2d - |4d - |r_x|||}{2d} \quad (19)$$

with ϵ_i : $i = 0, 1, 2$ quantized in 2 additional reliability bits $r_i[j]$; $j = 0, 1$.

III. Performances of the SC-DTTB system

A. Simulation results of 64-QAM with concatenated coding

The SC-DTTB system is a bandwidth limited system because the maximum channel bandwidth is determined at $B = 8$ MHz. In a bandwidth limited system it is in general not only the power requirements which determines the modulation and coding scheme that will be used. In the case of the SC-DTTB scheme it is a combination of a few constraints which decide what kind of system is used:

1. The commonality with the cable and satellite system.
2. Only a channel bandwidth of 8 MHz is available.
3. The required signal to noise ratio has to be as low as possible for covering a large array with a BER (Bit Error Ratio) better than 10^{-11}.

To satisfy constraint 2 it is very important to use 2^m-QAM spectral efficient modulation schemes i.e. 64-QAM, 128-QAM and 256-QAM. The spectral efficiency of uncoded 2^m-QAM is much worse than the possible spectral efficiency stated as the channel capacity and defined in [6]. This difference can be decreased by the use of powerful codes or coded modulation [3]. When the modulation is treated as a separate operation independent of the encoding, which is the case for the SC-DTTB system, the use of very powerful codes is required to offset the loss and provide some significant coding gain [3]. These powerful codes for the SC-DTTB system are a concatenation of Reed-Solomon $RS[204, 188, 17]$ coding (outer code) and convolutional coding (inner code) with code rates $R_{conv} = 1/2, 2/3, 3/4, 5/6$ and $7/8$. The decoding is performed by a Viterbi decoder and a Reed-Solomon $RS[204, 188, 17]$ decoder. This concatenated coding scheme also satisfies constraint 1 and constraint 3. The performances of 64-QAM with concatenated coding on an AWGN channel is shown in Fig.9 where the parameter R represents the code rate of the convolutional coding R_{conv} and with

$$SNR[dB] = \frac{E_s}{N_0}\,|_{dB}\,. \quad (20)$$

These performances are obtained through simulation (only hard decision) of the SC-DTTB system with ideal synchronization and extrapolation of the curves for BER $\leq 10^{-4}$.

From Fig.9, it can be seen that without inner coding (only Reed Solomon coding), the curve strongly bends behind a certain point. This is due to the fact that the Reed-Solomon coding has relative bad performance when the error rate is high, $BER \geq 10^{-3}$. Fig.9 shows also the increasing of the SNR by increasing the parameter R. The spectral efficiency as a function of R_{conv}

$$\eta_{QAM+conv.} = R_{conv}\, m\, \zeta\, 2 \quad (21)$$

increases when R_{conv} increases. This means there has to be made a trade off between power and spectral efficiency. For example, 64-QAM with $R_{conv} = 1/2$ and $R_{RS} = 188/204$ will decrease the spectral efficiency from 6 (no coding) to 2.77 but from Fig.9 can be seen that for this particular rate the $SNR \approx 21.5$ dB ($BER = 10^{-11}$) has the smallest value. The contradiction of, on one hand decreasing R_{conv} for less power requirements and on the other hand increasing R_{conv} for a better spectral efficiency has to be considered carefully by deciding what rate is used for the coding mechanism. The SC-DTTB system

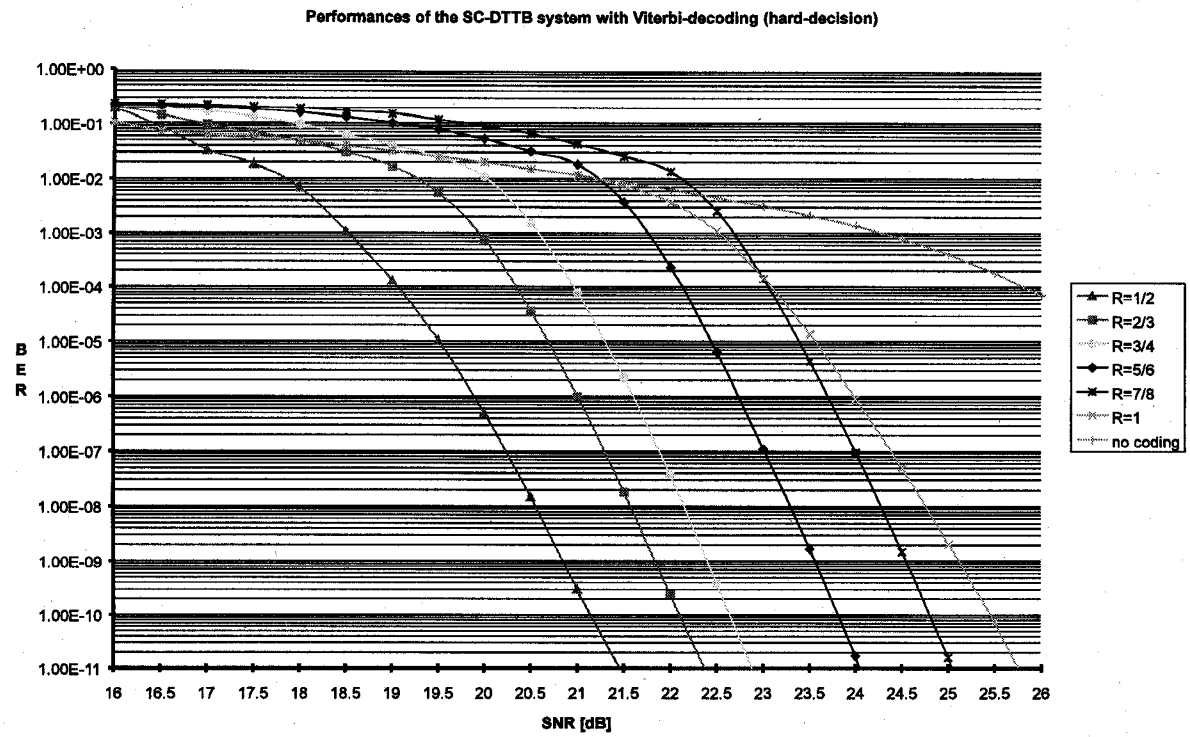

Fig. 9. 64-QAM with concatenated coding (only hard decision) on an AWGN channel.

has the possibility to operate at all the coding rates R shown in Fig.9. The power requirements, at the different code rates, to obtain a $BER = 10^{-11}$ are given by Table I. The maximum useful bit-rates which can be achieved with the SC-DTTB system are calculated by

TABLE I

The performances of the SC-DTTB system.

SC-DTTB system	SNR if $BER = 10^{-11}$
$R = 1/2$	≈ 21.5 dB
$R = 2/3$	≈ 22.4 dB
$R = 3/4$	≈ 22.9 dB
$R = 5/6$	≈ 24.0 dB
$R = 7/8$	≈ 25.0 dB
$R = 1$	≈ 25.8 dB

$$R_u = B \cdot R_{conv} \cdot \zeta \cdot m \cdot R_{RS} \tag{22}$$

and are represented by Table II.

Appendix

The conditional probability $P[m = \hat{m} \mid \underline{r}]$ (parameters defined in Fig.6) is a-posteriori probability, this means the probability of the symbols after they passed the channel also represented by $P[\underline{s}_i \mid \underline{r} = \underline{\rho}]$. The a-priori probability $P[\underline{s}_i]$ is the probability of the symbols before they pass the channel. The receiver makes a good guess to set $\hat{m} = m_k$ if and only if

TABLE II

The useful bit-rates of the SC-DTTB system.

SC-DTTB system	Useful bit-rates $\alpha = 0.15$
$R = 1/2$	$19.23 \cdot 10^6$ bits/s
$R = 2/3$	$25.64 \cdot 10^6$ bits/s
$R = 3/4$	$28.85 \cdot 10^6$ bits/s
$R = 5/6$	$32.05 \cdot 10^6$ bits/s
$R = 7/8$	$33.66 \cdot 10^6$ bits/s
$R = 1$	$38.47 \cdot 10^6$ bits/s

$$\begin{gathered} P[m_k|\underline{r} = \underline{\rho}] > P[m_i|\underline{r} = \underline{\rho}] \\ i = 0, 1, ..., L-1 \quad i \neq k. \end{gathered} \tag{23}$$

$P[m_i \mid \underline{r} = \underline{\rho}]$ can be rewritten with mixed Bayes rule [5]

$$\begin{aligned} P[m_i|\underline{r} = \underline{\rho}] &= \frac{P[m_i, \underline{r} = \underline{\rho}]}{P[\underline{r} = \underline{\rho}]} \\ &= \frac{P[m_i] \cdot P[\underline{r} = \underline{\rho}|m_i]}{P[\underline{r} = \underline{\rho}]} \\ &= \frac{P[m_i] \cdot p_{\underline{r}}(\underline{\rho}|m_i)}{p_{\underline{r}}(\underline{\rho})} \end{aligned} \tag{24}$$

Since, $m = m_i$ implies the event $\underline{s} = \underline{s}_i$ and vice versa

$$P[m_i|\underline{r} = \underline{\rho}] = \frac{P[m_i] \cdot p_{\underline{r}}(\underline{\rho}|\underline{s} = \underline{s}_i)}{p_{\underline{r}}(\underline{\rho})} \tag{25}$$

$p_{\underline{r}}(\underline{\rho})$ is independent of i, so the optimum receiver, on observing $\underline{r} = \underline{\rho}$, set $\hat{m} = m_k$ whenever the decision function

$$P[m_i] \cdot p_{\underline{r}}(\underline{\rho}|\underline{s} = \underline{s}_i) \quad , i = 0, 1, ..., L-1 \tag{26}$$

is maximum for $i = k$, this is the MAP receiver. A receiver that determines $\hat{m}$ only by maximizing

$$p_{\underline{r}}(\underline{\rho}|\underline{s} = \underline{s}_i) \quad , i = 0, 1, ..., L-1 \tag{27}$$

for all i, is called "a maximum likelihood receiver". It can be seen from (26) and (27) that the MAP receiver performs better than the ML receiver if the a-priori probability is not equal for every symbol. Because the MAP receiver takes the a-priori probability into account [5]. The received signal $\underline{r} = \underline{s} + \underline{n}$, where $\underline{n}$ denotes a zero mean white Gaussian process. Hence, (26) can be rewritten as

$$\begin{aligned} &P[m_i] \cdot p_{\underline{r}}(\underline{\rho}|\underline{s} = \underline{s}_i) = P[m_i] \cdot p_{\underline{n}}(\underline{\rho} - \underline{s}_i|\underline{s} = \underline{s}_i) \\ &i = 0, 1, ..., L-1 \end{aligned} \tag{28}$$

with $\underline{r} = \underline{\rho}$ and $\underline{s} = \underline{s}_i$. The signal $\underline{s}$ and the noise $\underline{n}$ are statistical independent, [5]

$$\begin{aligned} p_{\underline{n}}(\underline{\rho} - \underline{s}_i|\underline{s} = \underline{s}_i) &= \frac{p_{\underline{n}}(\underline{\rho} - \underline{s}_i, \underline{s} = \underline{s}_i)}{p_{\underline{n}}(\underline{s} = \underline{s}_i)} \\ &= \frac{p_{\underline{n}}(\underline{\rho} - \underline{s}_i) \cdot p_{\underline{n}}(\underline{s} = \underline{s}_i)}{p_{\underline{n}}(\underline{s} = \underline{s}_i)} \\ &= p_{\underline{n}}(\underline{\rho} - \underline{s}_i). \end{aligned} \tag{29}$$

Hence, the decision function of (28) is therefore

$$P[m_i] \cdot p_{\underline{n}}(\underline{\rho} - \underline{s}_i) \quad , i = 0, 1, ..., L-1 \tag{30}$$

The noise density function

$$p_n(\underline{\alpha}) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \cdot e^{-\frac{|\underline{\alpha}|^2}{2\sigma^2}} \tag{31}$$

is the density function of statistical independent, zero mean, Gaussian random variables with variance σ^2 and with N the number of vector components. This substitute in (30) yields

$$P[m_i] \cdot e^{-\frac{|\underline{\rho} - \underline{s}_i|^2}{2\sigma^2}} \quad , i = 0, 1, ..., L-1 \tag{32}$$

The optimum receiver sets $\hat{m} = m_k$ if (32) is maximum for $i = k$. The factor with 2π is discard because it is independent from i. To maximize (32) the exponent of

$$\left[e^{-\left\{|\underline{\rho} - \underline{s}_i|^2 - 2\sigma^2 \ln(P[m_i])\right\}}\right]^{\frac{1}{2\sigma^2}} \tag{33}$$

has to be minimized, (33) is the rewritten version of (32). Hence, maximizing (32) is equivalent to minimizing

$$|\underline{\rho} - \underline{s}_i|^2 - 2\sigma^2 \ln(P[m_i]) \tag{34}$$

for all values of i. The term $|\underline{\rho} - \underline{s}_i|^2$ is the square of the Euclidean distance between the points $\underline{\rho}$ and $\underline{s}_i$

$$|\underline{\rho} - \underline{s}_i|^2 = \sum_{j=1}^{N} (\rho_j - s_{ij})^2 \tag{35}$$

If every symbol m_i has equal a-priori probability then the optimum receiver sets $\hat{m} = m_k$ if and only if $\underline{\rho}$ is closer to point $\underline{s}_k$ than to any other possible signal. Hence, with symbols m_i with equal probabilities the boundary of a decision region is in the middle of the Euclidean distance between two adjacent points in a constellation diagram. With an unequal a-priori probability the decision regions are modified. Two adjacent points s_i and s_j are separated by the Euclidean distance $2d$ and have unequal a-priori probabilities $P[m_i]$ and $P[m_j]$. Assume s_i and s_j are in an one dimensional space ($N = 1$) and there are no more symbols. If $P[m_i] = P[m_j]$ then the boundary lies in the middle of the Euclidean distance at d. If $P[m_i] \neq P[m_j]$ then the boundary is moving towards the message with the lowest probability because when the message that is received, exactly lies in the middle due to noise it is better to decide for the message with the highest a-priori probability. Assume that the distance between the decision boundary and s_i is A and the distance between the decision boundary and s_j is B as showed by Fig.10. These

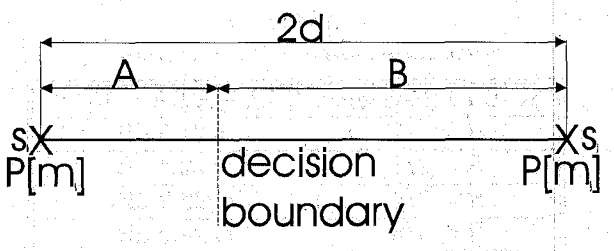

Fig. 10. The decision boundary of signals s_i and s_j.

distances can be calculated with

$$A^2 - 2\sigma^2 \ln(P[m_i]) = B^2 - 2\sigma^2 \ln(P[m_j]). \tag{36}$$

This can be rewritten as

$$\begin{aligned} A^2 &= B^2 - 2\sigma^2 (\ln(P[m_j]) - \ln(P[m_i])) \\ A^2 &= B^2 - 2\Delta \end{aligned} \tag{37}$$

with

$$\begin{aligned} \Delta &= \sigma^2 (\ln(P[m_j]) - \ln(P[m_i])) \\ &= \sigma^2 \ln\left(\frac{P[m_j]}{P[m_i]}\right). \end{aligned} \tag{38}$$

The distance from s_i to the decision boundary

$$A = |\underline{\rho} - \underline{s_i}| \tag{39}$$

plus the distance from s_j to the decision boundary

$$B = |\underline{\rho} - \underline{s_j}| \tag{40}$$

is the total distance $2d = A + B$ between s_i and s_j. When (37) is rewritten as

$$A^2 - B^2 = -2\Delta, \tag{41}$$

the parameters A and B can be given as function of Δ and d

$$A = d - \tfrac{\Delta}{2d} \quad B = d + \tfrac{\Delta}{2d}. \tag{42}$$

From (42) it can be seen that the decision boundary is shifted with $\frac{\Delta}{2d}$ towards the signal with the lowest a-priori probability $P[m_i]$. These distances are used to find the error probability $P[\varepsilon]$. The error probability $P[\varepsilon]$ is found immediately now the decision regions $\{I_i\}$ are determined. Once a good signal is received it belongs to the right decision region of $\{I_i\}$. Hence, if the signal is contaminated with additive white Gaussian noise

$$\begin{aligned} P[C|m_i] &= P[\underline{\mathrm{r}}\ in\ I_i|m_i] = \int_{I_i} p_{\underline{\mathrm{r}}}(\underline{\rho}|\underline{\mathrm{s}} = \underline{s_i}) d\underline{\rho} \\ &= \int_{I_i} p_{\underline{\mathrm{n}}}(\underline{\rho} - \underline{s_i}) d\underline{\rho} \\ &= \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \cdot \int_{I_i} e^{-\frac{|\underline{\rho}-\underline{s_i}|^2}{2\sigma^2}} d\underline{\rho}. \end{aligned} \tag{43}$$

In a one dimensional space (43) can be rewritten as

$$\begin{aligned} P[C|m_i] &= \frac{1}{\sqrt{2\pi}\sigma} \cdot \int_{I_i} e^{-\frac{|\rho-s_i|^2}{2\sigma^2}} d\rho \\ &= \frac{1}{\sqrt{2\pi}\sigma} \cdot \int_{-\infty}^{s_i+A} e^{-\frac{|\rho-s_i|^2}{2\sigma^2}} d\rho \\ &= \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\frac{A}{\sigma}} e^{-\frac{z^2}{2}} dz = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\frac{A}{\sigma}}^{\infty} e^{-\frac{z^2}{2}} dz \\ &= Q\left(-\frac{A}{\sigma}\right) = 1 - Q\left(\frac{A}{\sigma}\right) \end{aligned} \tag{44}$$

Hence, the conditional error probability

$$P[\varepsilon|m_i] = 1 - P[C|m_i] = Q\left(\frac{A}{\sigma}\right) = Q\left(\frac{d - \frac{\Delta}{2d}}{\sigma}\right) \tag{45}$$

and the conditional probability of signal s_j

$$P[\varepsilon|m_j] = 1 - P[C|m_j] = Q\left(\frac{B}{\sigma}\right) = Q\left(\frac{d + \frac{\Delta}{2d}}{\sigma}\right) \tag{46}$$

is determined. The overall probability

$$\begin{aligned} P[\varepsilon] &= \sum_{l=0}^{L-1} P[\varepsilon|m_l] \cdot P[m_l] \\ &= P[m_i] \cdot Q\left(\frac{d - \frac{\Delta}{2d}}{\sigma}\right) + P[m_j] \cdot Q\left(\frac{d + \frac{\Delta}{2d}}{\sigma}\right) \end{aligned} \tag{47}$$

The decision boundary is exactly at the middle between two adjacent symbols when the a-priori probabilities of the signals are equal i.e. $\Delta = 0$, so as with the 64-QAM constellation diagram.

If the a-priori probabilities of the signals are unequal, the decision boundary moves towards the signal with the lowest probability. The size of the movement $(\frac{\Delta}{2d})$ is a function of the Euclidean distance d, the variance σ^2 and the ratio of the a-priori probabilities given by (38).

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