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## Direct-Radiator Loudspeaker Systems with High-BI

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### ABSTRACT

This paper is an extension of AES 113th paper #5651 which shows additional consequences of a dramatic increase in the motor strength BI of a driver. Not only is the efficiency of the loudspeaker and amplifier greatly increased, but high BI-values have a positive influence on other aspects of loudspeaker systems. Box volume can be significantly reduced and other parameters can be altered. A prototype driver unit is studied which performs well in a small sealed box. Vented or passive radiator systems do not benefit as much from high BI.

### 1. Introduction

Direct-radiator loudspeakers typically have a very low efficiency, since the acoustic load on the diaphragm or cone is relatively low compared to the mechanical load, and in addition the driving mechanism of a voice coil is quite inefficient in converting electrical energy into mechanical motion. The drivers have a magnetic structure that is deliberately kept mediocre so that the typical response is flat enough to use the device without significant equalization.

Some decade ago a new rare-earth-based material, neodymium-iron-boron (NdFeB) in sintered form, came into more common use. It has a very high flux density coupled with a high coercive force, possessing a B-H product up by almost an order of magnitude. This allows drivers to be built in practice

with much larger total magnetic flux, thereby increasing BI by a large factor. An earlier paper of ours has outlined some features of normal sealed-box loudspeakers with greatly-increased BI [1]. That work focused mainly on efficiency of the system as applied to several amplifier types, but also indicated several other avenues of interest.

This paper is a study by simulation and some experiments of (a) sealed loudspeakers with modified box sizes and driver parameters, and (b) vented loudspeakers with dramatically increased BI values. We focus on the bass and mid portion of the loudspeaker frequency range, since this is the region of greatest challenge. A glance at some products available today suggests that at least some of the loudspeaker industry is aware of these ideas. In conversation with industry practitioners, we found that much of what we learned is well known to the

community, although there has not been much public disclosure.

Again we will find that equalization *must* be used. The acoustic efficiency does increase a lot, and much smaller box sizes are possible. Increased cone masses provide interesting characteristics as well. For vented systems of high BL, the port resonance is ill matched to an over-damped box resonance, making vented systems unsuitable at high BL.

Earlier work [2] more than a decade ago considered a loudspeaker with a moderately-high BL value, coupled with a class-G amplifier, so that the increased efficiency of the driver complemented the efficiency of the amplifier. The study did not give detailed information on the signal statistics or loudspeaker performance, but dealt mainly with the amplifier technology. Even earlier work at Philips [3] dealt with much of the field of loudspeakers and the relationships between parameters.

## 2. Sealed-Box Loudspeaker Model

We first reiterate briefly the theory for the sealed loudspeaker as presented in [1], then follow this later on by the addition of a port to represent a vented system. Although we employed the full complex acoustic radiation impedance for the driver in that earlier paper, at the lower frequencies for which we use the theory this was not really necessary. In what follows we use a driver model with a simple acoustic air load. Beranek [4] shows that for a baffled piston this air load is a mass of air equivalent to  $0.85a$  in thickness on each side of a piston of radius  $a$ . In fact the air load can exceed this value, since most drivers have a support basket which obstructs the flow of air from the back of the cone, forcing it to move through smaller openings. This increases the acceleration of this air, augmenting the acoustic load.

The driver is characterized by a cone or piston of area  $S_C = \pi a^2$ , a force factor  $Bl$ , electrical coil resistance  $R_E$ , total moving mass  $M$  (which includes the air load), mechanical damping coefficient  $b$ , and suspension spring constant  $k_S$ , defined in terms of free-air resonance frequency  $f_0$  by  $k_S = (2\pi f_0)^2 M$ . The sealed box of volume  $V_0$  has a restoring force on the piston with equivalent spring constant  $k_B = \gamma p_0 S_C^2 / V_0$ , where  $\gamma = 1.4$  for air, and  $p_0$  is atmospheric pressure.

The current  $i(t)$  taken by the driver when driven with a voltage  $v(t)$  will be given by equating that voltage source to the ohmic loss and the induced voltage:

$$v(t) = i(t) R_E + Bl \, dx_C/dt,$$

where  $x_C$  is the piston displacement. The term in  $dx_C/dt$  is the voltage induced by the driver piston velocity of motion. We ignore the inductance of the loudspeaker and the effect of the Eddy currents induced in the pole structure [6]. For harmonic signals described by  $e^{j\omega t}$ , the previous equation becomes (using capitals for variables in the frequency domain):

$$V(\omega) = I(\omega) R_E + j\omega Bl X_C(\omega). \quad (1)$$

The external electromagnetic force  $Bl I(\omega)$  is related by Newton's law to the inertial reaction and all the other forces by

$$Bl I(\omega) = (-\omega^2 M + j\omega b + k_S + k_B) X_C(\omega). \quad (2)$$

By eliminating  $I(\omega)$ , Eqs. (1) and (2) give a final relationship between  $X_C(\omega)$  and  $V(\omega)$ :

$$X_C(\omega) = \frac{(Bl/R_E)V(\omega)}{(-\omega^2 M + j\omega\{b + (Bl)^2/R_E\} + k_S + k_B)}. \quad (3)$$

We use an infinite baffle to mount the piston ( $2\pi$  loading), and in the compact-source regime the far-field acoustic pressure  $p(t)$  a distance  $r$  away becomes

$$p(t) = \rho S_C (d^2 x_C / dt^2) / (2\pi r),$$

proportional to the volume acceleration of the source [4,5]. In the frequency domain we have

$$P(\omega) = -\omega^2 \rho S_C X_C(\omega) / (2\pi r). \quad (4)$$

The compact source regime, in which  $a \ll \lambda / (2\pi)$ , does not apply at higher frequencies, and the speaker starts to beam acoustic radiation. We choose to extend Eq. (4) to apply over the whole frequency domain. In this way a mass-controlled model gives a nominally flat response over the whole audio band. In reality we would at high frequencies cross over the system to a midrange or tweeter driver, but we do not discuss that here.

By substituting Eq. (3) into Eq. (4), we get the usual frequency response  $H(\omega)=P(\omega)/V(\omega)$ :

$$H(\omega) = \frac{-\omega^2 \rho (S_C / 2\pi r) (Bl/R_E)}{(-\omega^2 M + j\omega \{b + (Bl)^2/R_E\} + k_S + k_B)}. \quad (5)$$

Eqs. (2) and (3) can be solved for the current  $I(\omega)$  in terms of  $V(\omega)$ , the voltage input to the loudspeaker. This driving voltage is usually supplied by an amplifier of low output impedance. The electrical impedance of the loudspeaker can be calculated as  $Z(\omega) = V(\omega)/I(\omega)$ :

$$Z(\omega) = \frac{R_E (-\omega^2 M + j\omega \{b + (Bl)^2/R_E\} + k_S + k_B)}{(-\omega^2 M + j\omega b + k_S + k_B)}. \quad (6)$$

Note that as  $Bl$  increases, the damping term  $(Bl)^2/R_E$  dominates the behaviour. For large  $Bl$ ,  $Z(\omega)$  will be reactive, with phase very close to  $+90^\circ$  or  $-90^\circ$ . The inverse of the impedance, the admittance  $Y(\omega)=I(\omega)/V(\omega)$ , can be used to calculate the current waveform when audio signals are applied to the loudspeaker. We refer the reader to [1] in which plots of  $H(\omega)$  and  $Z(\omega)$  are given, and an analysis of the large increase in efficiency when  $Bl$  is increased.

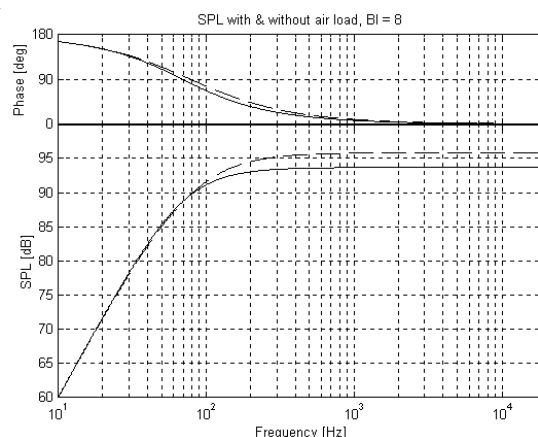
### 3. Sealed-Box Calculations and Experiment

Figure 1 shows two frequency responses of the model loudspeaker, using Eq. (5) with parameters typical of an 8-inch loudspeaker having a voice coil diameter of about 2 cm. The curves show the effect of the air load on a driver. The thin line is the response when the air load  $m_A$  of 3.28 g is not included.

$$\begin{aligned} M &= 0.015 \text{ kg (total moving mass)} \\ a &= 0.08 \text{ m} \\ m_A &= 0.00328 \text{ kg (acoustic air load)} \\ Bl &= 8.0 \text{ N/A} \\ R_E &= 6.0 \text{ ohms} \\ f_0 &= 30.0 \text{ Hz} \\ b &= 1.0 \text{ N}\cdot\text{s/m} \\ V_0 &= 0.025 \text{ m}^3 \\ r &= 1.0 \text{ m.} \end{aligned}$$

The air load has decreased the response by about 2 dB due to the added mass. The damping is determined largely by  $Bl$ , and partly by  $b$ , and has

been chosen to give an approximate Butterworth high-pass characteristic, so that there is no peak in the bass.



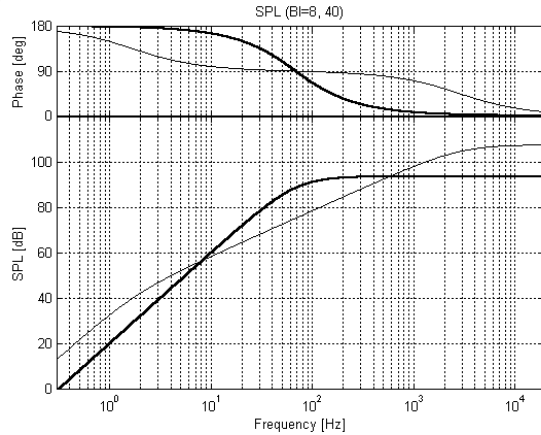
**Figure 1.** Frequency response of the loudspeaker model defined in the text for  $Bl=8.0$  N/A. The solid line shows the response including the acoustic air load, while the dashed line is the response without the acoustic load. The air load near the baffled piston makes a significant difference to the response.

For these responses the input voltage to the loudspeaker is  $\sqrt{8}$  volts, and the output sound pressure is normalized to  $2 \times 10^{-5}$  Pa, so the curves give the sound pressure level (SPL) at 1 m for 1 watt into a nominal 8 ohm load. We have used an infinite baffle to mount the transducer ( $2\pi$  loading), so the response will be down by 6 dB if the unit is mounted away from any boundaries ( $4\pi$  loading). This would reduce the sensitivity of the system to about 87 dB SPL, a value typical for the parameters listed. Other parameters are atmospheric pressure  $p_0=1.013 \times 10^5$  Pa, speed of sound  $c=343$  m/s, adiabatic constant  $\gamma=1.40$ , and density of air  $\rho=1.20$  kg/m<sup>3</sup>.

### 4. The Effect of High Bl

Figure 2 shows the frequency response curves Eq. (5) for two  $Bl$  values, 8.0 N/A and 40 N/A. At the higher  $Bl$  value the electromagnetic damping is very high. If we ignore the small mechanical and acoustic damping of the driver, the damping is proportional to  $(Bl)^2/R_E$ . For a Butterworth response, the inertial term  $\omega^2 M$ , damping  $\omega (Bl)^2/R_E$  and total spring constant  $k_S+k_D$  are all about the same at the bass cutoff frequency. When  $Bl$  is increased by a factor of 5, the damping is increased by a factor of 25. Thus the inertial factor, which must dominate at high

frequencies, becomes equal to the damping at a frequency about 25 times higher than the original cutoff frequency. This causes the flat response of the system to have a 6-dB/octave rolloff below that frequency, as shown in the figure.



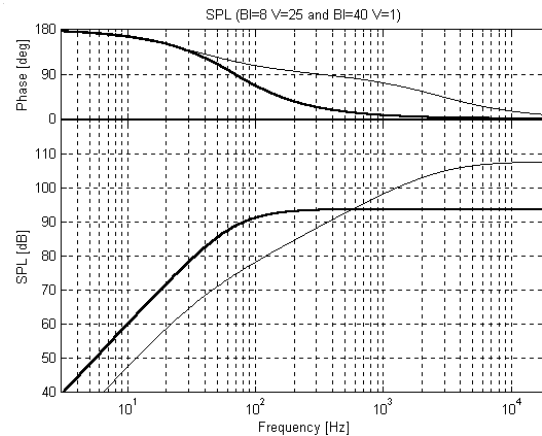
**Figure 2.** Frequency response curves of the loudspeaker model for BI=8.0 N/A (thick line) and for BI=40.0 N/A (thin line).

At very low frequencies, the spring restoring force becomes important relative to the damping force at a frequency 25 times lower than the original cutoff frequency. Below this, Figure 2 shows that the rolloff is 12 dB/octave. Such frequencies are too low to influence audio performance, but it is clear that the box is now no longer constraining the low-frequency performance. We could use a much smaller box without serious consequences!

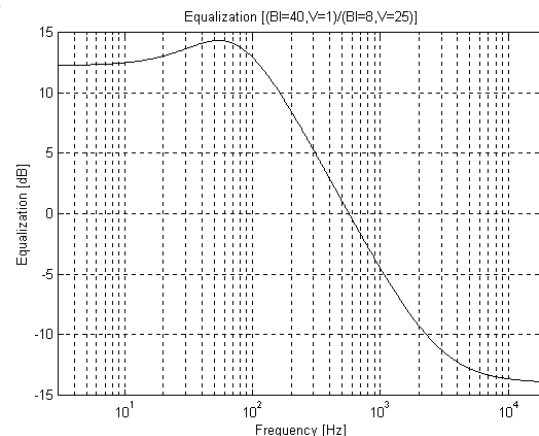
How much smaller can the box be? The low-frequency breakpoint has been moved down by a factor of nearly 25. Suspension stiffness  $k_s$  is small, and since  $k_B \propto (1/Vol)$ , the breakpoint will return to the initial bass cutoff frequency when the box size is reduced by a factor of about 25! Our 25-L box could be reduced to 1 L! This appealing aspect of high BI is shown in Figure 3. The thin curve in the figure shows the breakpoint between the 12- and 6-dB/octave slopes at about 40 Hz. The smaller box has increased this frequency from very low values. Powerful electrodynamic damping has allowed the box to be reduced in volume without sacrificing the response at audio frequencies! The only penalty is that we must apply some equalization.

The equalization needed to restore the response to the original value is shown in Figure 4. The slope is close to 6 dB/octave. The break-point geometry of

Figure 3 shows that this results in a curve that has essentially the BI-ratio of 5 (+14 dB) increase at the original cutoff frequency, but reducing at high frequencies to an attenuation of 5 (-14 dB). Such equalization will in virtually all cases increase the voltage excursion applied to the loudspeaker, as shown in [1], since audio energy resides principally at lower frequencies. The curve levels out at just over 12 dB at low frequencies, but in actual use one might attenuate frequencies below, say, 40 Hz.



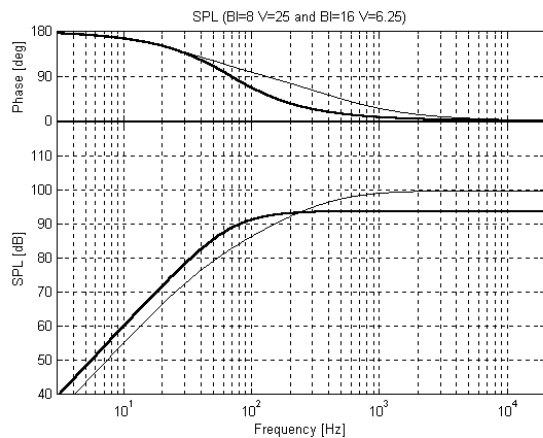
**Figure 3.** Frequency response curves of the loudspeaker model for a box size of 25 L and BI=8.0 N/A (thick line) and for a box size of 1.0 L and BI=40.0 N/A (thin line). The high BI value allows the use of very small boxes!



**Figure 4.** Equalization needed to restore the response to the original value after changing the box size from 25 L to 1.0 L, and increasing the BI value from 8.0 N/A to 40.0 N/A.

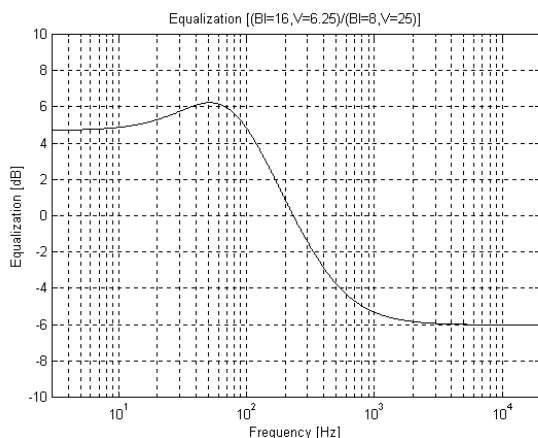
The example mentioned is perhaps far too optimistic. It would be very difficult to change BI by a factor of

5 without a total redesign of the driver. But changes of a factor of 2 are reasonable, and could be made by changing the parameters of the magnet and pole structure, without much change in cone mass or other characteristics. Figure 5 shows the output of a system whose 25-L box has been reduced by a factor of four to 6.25 L while the BL is doubled.



**Figure 5.** Frequency response curves of the loudspeaker model for box size of 25 L and BL=8 N/A (thick line) and for box size of 6.25 L and BL=16 N/A (thin line).

Note that the breakpoint of the system between 6 and 12 dB/octave output is at a frequency of about 50 Hz. The box spring is thus not limiting the bass, but of course equalization must still be applied, as shown in Figure 6. Again the equalization could be reduced below, say, 40 Hz.



**Figure 6.** Equalization needed to restore the response to the original value after changing the box size from 25 L to 6.25 L, and increasing the BL value from 8 N/A to 16 N/A.

The foregoing ideas cannot readily be tested since it is difficult to make drivers with precisely the characteristics we have outlined. However, we found a driver with a high BL that corroborates some of our conclusions. This driver is a 10-inch nominal bass/midrange driver (26 cm) prototype for a professional sound reinforcement system. It has a total moving mass of 56 g, cone diameter of 21 cm, free-air resonance of 41.4 Hz, dc resistance of 7.5 ohms, and a BL of 22 N/A!

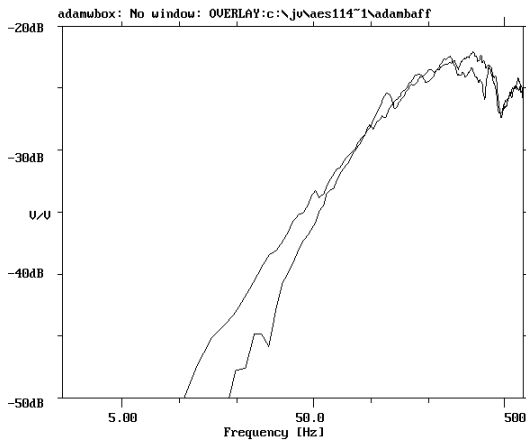
The relative damping (discussed in section 8) for this driver is 4.43, which is 3.13 times that for a nominal Butterworth unit aligned to 41.4 Hz. In free air the impedance has a maximum of 540 ohms! This alone indicates a high BL-value and/or low suspension damping, as can be seen from Eq. (6), since the impedance magnitude will be  $R_E + (BL)^2/b$  at resonance. The unit has a mass of 8kg! Its magnet structure is 19 cm in diameter and the top plate is 1.2 cm thick. A photograph of the driver is shown in Figure 7.



**Figure 7.** Photograph of the high-BL driver. The device is 26 cm in diameter and the massive magnet structure is 19 cm in diameter.

Figure 8 shows the nearfield response of this driver in two different situations. The upper curve is the response at the dustcap of the unit when mounted in a baffle. This ensures a normal air load which will be similar to that when the unit is mounted in a box. The lower curve is the nearfield response when the driver is mounted in a 9-L box. This box seems

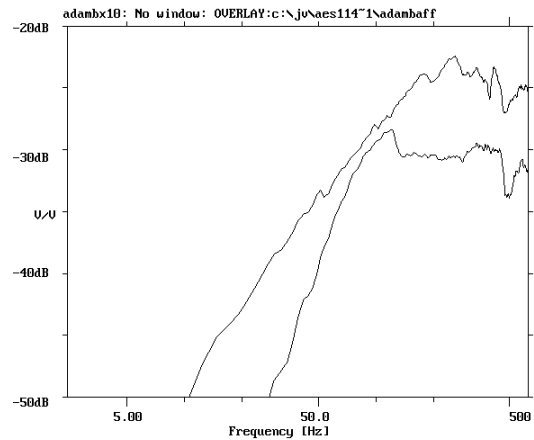
ridiculously small given the size and weight of the driver, but we shall see that it is appropriate.



**Figure 8.** Nearfield frequency response of high-BL prototype driver unit. Upper curve is reference response when mounted in a baffle, lower when mounted in a 9-L box. The high damping reduces the low-frequency reduction from such a small box.

Although the measured resonance frequency on the box is 83 Hz, the output is down by 3 dB from the baffled response only below 30 Hz! The strong magnet structure makes the response at low frequencies rise at 6 dB/octave, which must be equalized out of course. At higher frequencies the effects of cone breakup and other resonances are visible.

To see what would happen if BL were decreased, we could place a resistor in series with the loudspeaker in order to reduce the  $(BL)^2/R_E$  damping term. Figure 9 shows the effect of measuring the loudspeaker, mounted in the 9-L box, but with a 10-ohm series resistor. This reduces the effective damping by a factor  $7.5/(7.5+10)$ . Note that the response rapidly diminishes just below 80 Hz, which is the resonance frequency on the box. With this increased resonance frequency, the actual damping is just a bit below a Butterworth alignment, as our theory predicts. Thus the system now displays a slightly underdamped response.



**Figure 9.** Nearfield frequency response of high-BL prototype driver unit. Upper curve is reference response when mounted in a baffle, lower when mounted in a 9-L box and measured with a 10-ohm series resistor. The damping is reduced by a factor of 2.3, and now the 83 Hz box resonance frequency reduces the response by 12 dB/octave below that.

## 5. An Observation about Equalization

In [1] and for the earlier sections of this paper, we calculated the required equalization by calculating the frequency response ratio  $H_L(\omega)/H_H(\omega)$  using Eq. (5), where the subscripts refer to the high and low values of BL. The required equalization function for two loudspeakers with different BL, but identical in other respects, can also be calculated using an alternative approach, which gives new insight.

For our two loudspeakers to produce the same acoustic output, *the shape and motion* of the two pistons or cones must be the same. Since all other aspects of the loudspeakers are the same, this can be achieved if the *total force* on the pistons is the same, thus ensuring that they have the same motion. The instigating force is derived from the electromagnetic Lorentz force,  $BL I(\omega)$ . If this force is the same in the two cases, then the responding forces such as the pressure inside the box, the acoustic radiated pressure, or other mechanical forces, will be the same. Thus the *total piston force* will be the same.

Since current  $I(\omega) = V(\omega)/Z(\omega)$ , where  $V(\omega)$  is the loudspeaker voltage and  $Z(\omega)$  is its electrical impedance, we must arrange to have  $BL V(\omega)/Z(\omega)$  the same for the two conditions. Hence

$$BL_H V_H(\omega)/Z_H(\omega) = BL_L V_L(\omega)/Z_L(\omega).$$

Note however that since  $H_{EQ}(\omega) = V_H(\omega)/V_L(\omega)$ , then

$$H_{EQ}(\omega) = [Bl_L/Z_L(\omega)]/[Bl_H/Z_H(\omega)], \quad (7)$$

a very simple relationship that indicates the importance of the electrical impedance in determining loudspeaker characteristics. We can verify the result from Eqs. (5) and (6). Note that it applies for the response at any orientation, not just on axis, and represents a general property of acoustic transducers with magnetic drive.

## 6. Vented-Box Loudspeaker Model

To model the vented or ported loudspeaker we consider the driver mounted in a box that has a port with its opening nearby on the baffle. This model has been studied long ago by Thiele [7] and Small [8]. Here we use a physical approach with explicit driver and other acoustic parameters.

A port of area  $S_p$  and length  $L$  has a mass of air  $m = \rho L S_p$ , and we think of this plug of air as moving in response to the pressure inside the box. The displacement  $x_p(t)$  of this air causes a change in pressure inside the box, which causes a force on the piston. The motion of the piston and the air in the port are thus coupled by the spring resulting from the air in the box.

When we apply Newton's Law  $Ma = \Sigma F$  to the piston including the external electromagnetic force  $Bl I(\omega)$ , the inertial and all the other forces are related by:

$$-\omega^2 M X_C(\omega) = -j\omega b X_C(\omega) - (k_S + k_B) X_C(\omega) - k_B (S_p/S_C) X_p(\omega) + Bl I(\omega). \quad (8)$$

The second line contains the force on the piston caused by the motion of the air in the port. It is convenient to use the spring constant of the box for the piston,  $k_B$ , normalized by the port/piston area ratio, to represent this force.

By eliminating  $I(\omega)$  from Eqs. (1) and (8), the final relationship between  $X_C(\omega)$ ,  $X_p(\omega)$  and  $V(\omega)$  is:

$$\begin{aligned} [-\omega^2 M + j\omega\{b + (Bl)^2/R_E\} + k_S + k_B] X_C(\omega) \\ + k_B (S_p/S_C) X_p(\omega) \\ = N_1(\omega) X_C(\omega) + k_B (S_p/S_C) X_p(\omega). \end{aligned}$$

$$= Bl V(\omega)/R_E. \quad (9)$$

$N_1(\omega)$  represents the quadratic factor in  $\omega$  in square brackets in the first line of the equation, and has the dimensions of a spring constant. It is positive at low frequencies. Note that the factor  $j\omega (Bl)^2/R_E$  is the electromagnetic damping. It is usually much higher than the damping due to the acoustic impedance (which we have neglected) or the damping coefficient  $b$ .

The acceleration of the air mass  $m$  in the port is related by Newton's law to the total pressure in the box, which in turn relates to the displacements of both piston and port, and the port damping coefficient  $\beta$ :

$$\begin{aligned} -\omega^2 m X_p(\omega) &= p_{Box} S_p \\ &= (\gamma p_0/V_0)(-S_C X_C - S_p X_p) S_p - j\omega \beta X_p(\omega). \end{aligned}$$

Solving for  $X_C$  gives:

$$\begin{aligned} X_C(\omega) &= [(\omega^2 m - j\omega \beta) V_0 / (\gamma p_0 S_p S_C) - S_p / S_C] X_p(\omega) \\ &= N_2(\omega) X_p(\omega). \end{aligned} \quad (10)$$

$N_2(\omega)$  is also a quadratic resonance factor in  $\omega$ , and it is dimensionless. This term is negative at low frequencies, meaning that the motion of the cone and the air in the port is in antiphase, as we would expect.  $\beta$  is usually very small and the port resonance is then essentially undamped, with  $X_p(\omega)$  being very large relative to  $X_C(\omega)$  near the resonance frequency given by  $\omega^2 = \gamma p_0 S_p^2 / (m V_0)$ . There will however be some damping due to coupling with the cone assembly. By substituting Eq. (10) into Eq. (9), we can obtain a result for either  $X_C(\omega)$  or  $X_p(\omega)$ .

If we choose to eliminate  $X_p(\omega)$  we get:

$$[N_1(\omega) + k_B S_p / (S_C N_2(\omega))] X_C(\omega) = Bl V(\omega)/R_E, \quad (11)$$

while eliminating  $X_C(\omega)$  gives:

$$[N_1(\omega) N_2(\omega) + k_B (S_p/S_C)] X_p(\omega) = Bl V(\omega)/R_E. \quad (12)$$

Rather than expanding Eqs. (11) and (12) to display all the terms, we will use the definitions of  $N_1(\omega)$  and

$N_2(\omega)$  in MATLAB to work out  $X_C(\omega)$  and  $X_P(\omega)$  from these equations. To appreciate the nature of these responses we can study the  $\omega$ -behaviour of the resulting equations.

In Eq. (11), the term in square brackets containing  $N_1$  and  $N_2$  will be fourth order in  $\omega$  in the numerator, and second order in the denominator, including a constant term. Hence  $X_C(\omega)$ , proportional to the reciprocal of that term, is also a fourth-order response, low-pass in nature, but asymptotically having a second-order rolloff  $1/f^2$  at high frequencies. This is reasonable since the piston inertia will dominate at high frequencies. At low frequencies the response will be non-inverting, as would be expected for the forced motion of a simple spring, described by the suspension spring constant  $k_s$ .

The term in square brackets in Eq. (12) is a quartic in  $\omega$ . Thus  $X_P(\omega)$  is a fourth-order low-pass response. The response at low frequencies is inverting, as would be expected since slow motion of the piston will cause airflow in the port in the opposite direction. At the highest frequencies the piston response goes as  $1/f^2$ . This is the driving force for the air in the port, whose inertia will make its response fall by another factor  $1/f^2$ , leading to a  $1/f^4$  factor.

As earlier, we use an infinite baffle ( $2\pi$  loading) to mount the piston and the port, and in the compact-source regime the far-field acoustic pressure  $p_V(t)$  a distance  $r$  away is proportional to the sum of the volume acceleration [5] from the piston and the port:

$$p_V(t) = \rho [S_C(d^2x_C/dt^2) + S_P(d^2x_P/dt^2)]/(2\pi r).$$

In the frequency domain we have:

$$P_V(\omega) = -\omega^2 \rho [S_C X_C(\omega) + S_P X_P(\omega)]/(2\pi r). \quad (13)$$

By substituting for  $X_C(\omega)$  and  $X_P(\omega)$  in terms of  $V(\omega)$  into Eq. (13), we get the usual frequency response for the vented system:

$$H_V(\omega) = P_V(\omega)/V(\omega). \quad (14)$$

The motion of the air in a port occurs mainly near the speaker resonance frequency. At this low frequency the imaginary part of the acoustic impedance is

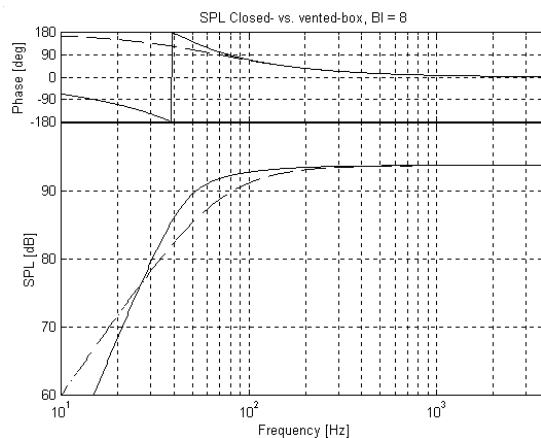
dominant, representing an extra mass added to the mass of the air in the port. This can be absorbed into the port length  $L$ , which is the physical length of the port plus end corrections. For a flanged end, we add  $0.85r$  to the length, and  $0.613r$  for an unflanged end [4], where  $r$  is the port radius. Thus even a port consisting of a round hole in a thin baffle (flanged both ends) has a length of  $1.70r$ .

## 7. Vented-Box Numerical Calculations

Many loudspeaker systems use ports or passive radiators to increase the response near the resonance frequency of the box. Does a high BI-value have a beneficial effect for a driver with a port? Figure 10 shows frequency responses of our earlier sealed model loudspeaker, but now compared to a system having an added port, having the length and area parameters below:

$$\begin{aligned} BI &= 8.0 \text{ N/A} \\ L &= 0.1 \text{ m} \\ S_P &= 0.00125 \text{ m}^2. \end{aligned}$$

Properly designed vented loudspeakers have approximately a fourth-order Butterworth high-pass response.



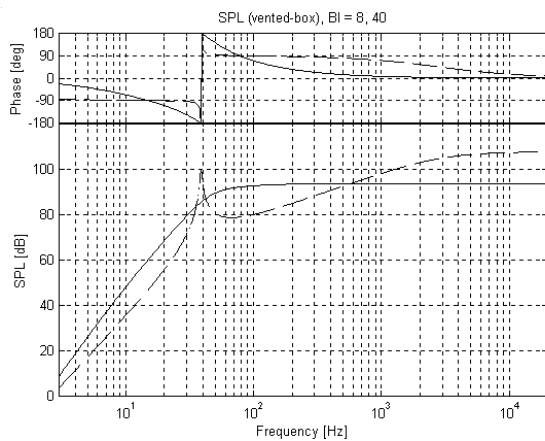
**Figure 10.** Frequency response of the vented loudspeaker model defined in the text for  $BI=8.0$  N/A (solid line), together with a sealed box loudspeaker having similar parameters (dashed line). At low frequencies the vented system has a 4<sup>th</sup>-order roll-off, while the sealed system has a 2<sup>nd</sup>-order roll-off.

Although somewhat unrealistic, to demonstrate clearly the effects with a port, we will consider a change in BI by a factor of 5. Figure 11 shows the



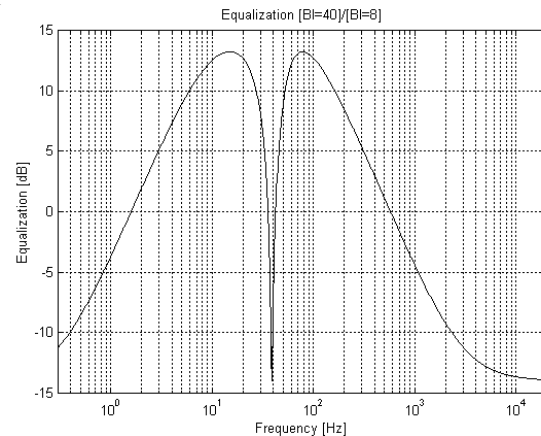
frequency response curves for the vented loudspeaker for two BI values, 8.0 N/A and 40 N/A. At the higher BI value the electromagnetic damping is very high. As for a sealed-box loudspeaker, a properly- designed vented loudspeaker also has the inertial term  $\omega^2 M$ , damping  $\omega (BI)^2/R_E$  and total spring constant  $k_S+k_D$  all about the same at the bass cutoff frequency. Again an increase in BI by a factor of 5 results in breakpoints for the inertial factor which move up in frequency by a factor of 25, and a spring factor breakpoint 25 times lower than the original frequency. Below this, the rolloff is 12 dB/octave as shown. These latter frequencies are too low to influence audio performance.

However, the resonance of the port on the box does not scale with BI. Also, higher BI heavily damps the cone, thereby decreasing the damping of the port resonance. This resonance is actually sharper than shown on the diagram; its peak goes to about 112 dB. Thus the equalization that is required, shown in Figure 12, will be quite impractical. Any small change in resonance caused by temperature or port variations would misalign the physical system and the electronic equalization.



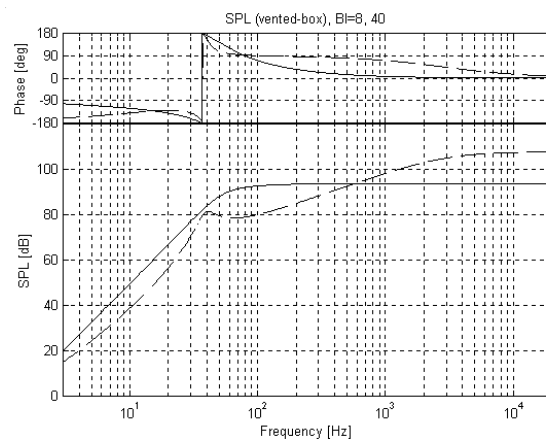
**Figure 11.** Frequency response curves of the vented loudspeaker model for BI=8.0 N/A (solid line) and for BI=40.0 N/A (dashed line). At high BI the port resonance is almost undamped.

The port damping is actually decreased by a high BI, since the cone is now virtually clamped by the electromagnetic braking. This makes the equalization very dependent on the precise resonance frequency of the port. Note that precisely at the port resonance, the required EQ has the same value as at very high and low frequencies. This is true because the cone has almost no output at these frequencies.



**Figure 12.** Equalization needed to restore the response to the original value after increasing the BI value from 8.0 N/A to 40.0 N/A. The port resonance becomes very sharp at high BI.

To avoid the very sharp port resonance, we might choose to damp the air motion in the port by introducing a flow resistance, for example, by placing a specific fabric material across the port entrance. Figure 13 shows the effect of a small port damping coefficient of 0.01. This has little effect on the normal BI=8 case, since the coupling of the port to the cone via the box air spring damps the port resonance much more than the added damping.

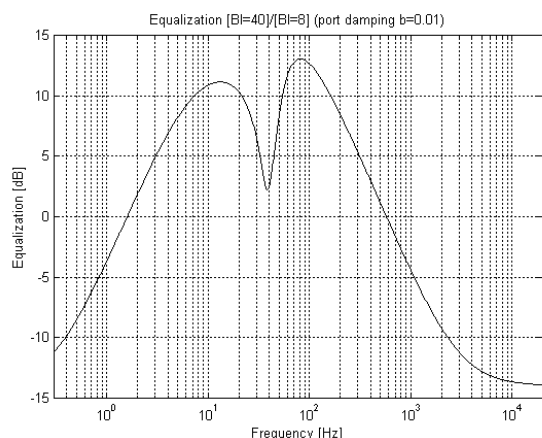


**Figure 13.** Frequency response curves of the vented loudspeaker model for BI=8.0 N/A (solid line) and for BI=40.0 N/A (dashed line), for a port damping coefficient of 0.01. This damps the port resonance somewhat, making equalization less troublesome.

The equalization required when modest port damping is employed is shown in Figure 14. This is a tolerable curve to implement and will not be so affected by parameter variations. Although we do not show it, lower BL values also prevent the port resonance from becoming too narrow.

When a high BL value is used, we have seen how sealed loudspeakers can employ much smaller boxes. This is not really possible with vented systems, since small boxes will require very long ports with small area in order to resonate at frequencies around 50 Hz, and such ports will exhibit viscous air losses and nonlinearity.

Whether a port is useful when BL is significantly increased is left to the reader to judge. There is some enhancement of the lowest frequencies, but the problem of alignment, the extra excursion caused by infrasonic inputs, and the possible requirement of a rather unusual damping militate against this approach. Intermediate BL values may still leave the port having some beneficial effect.



**Figure 14.** Equalization needed (after introducing modest port damping) to restore the response to the original value after increasing the BL value from 8.0 N/A to 40.0 N/A. Alignment problems are reduced since the port resonance is wider.

## 8. A Dimensionless Measure of Damping

What would be a good dimensionless parameter to describe the relative damping due to BL? As BL increases, the box and suspension restoring forces become less relevant, as we shall see, so we choose a parameter of the form

$$[j\omega (BL)^2/R_E]/[-\omega^2 M],$$

which is the ratio of the electrical damping force to the inertial force on the total mass  $M$  representing the cone with its air load. The unit imaginary and the negative sign should be removed, so the relative damping factor  $\delta$  becomes:

$$\delta = (BL)^2/[\omega_0 M R_E]. \quad (15)$$

The frequency  $\omega_0$  can be chosen to represent the low-frequency end of the intended audio spectrum, or it could be set to a reference frequency such as 50 Hz. The latter may be useful since the low-frequency cutoff of a system is significantly altered when BL is significantly increased. Incidentally, for the usual Butterworth system aligned to frequency  $\omega_0$ ,  $\delta$  would be  $\sqrt{2}$ . The driver studied in section 4 has  $\delta=4.43$ .

The common parameter  $Q_E$ , the electrical q-factor, while similar to  $1/\delta$ , is predicated on a normal driver for which the resonance frequency is the interaction between inertial and suspension forces. As BL is increased, the suspension forces are less relevant, and Eq. (15) is a better measure.

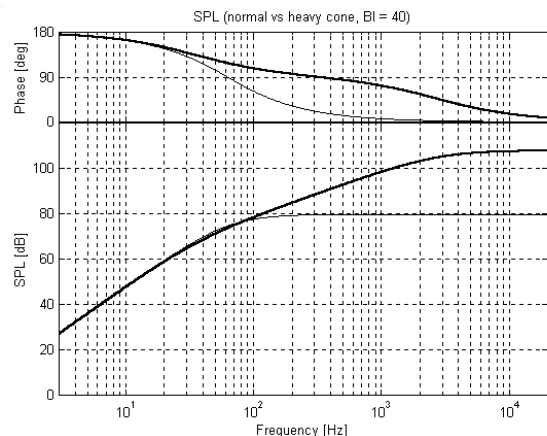
## 9. Other Aspects of High BL

Our earlier study with reduced box sizes displayed excellent bass characteristics due to the high BL and appropriate equalization. The cone inertial mass term has a breakpoint with the damping at very high frequencies such as several kHz. What would happen if we increase the mass of the cone, in order to force this breakpoint frequency back to normal?

Figure 15 shows two responses, both with a high  $BL=40$  N/A and a very small box size of 1.0 L. The thick curve is for a normal total moving mass of 15 grams. The box spring constant has a breakpoint to the damping slope at about 50 Hz, and the inertial term has a breakpoint to the damping slope at about 3 kHz. When the moving mass is increased 25 times to 375 grams, the breakpoints line up at about 50 Hz, and the output shown in the thin curve is nominally flat and would not require equalization.

Is the system useful? It can act as a bass system without any added equalization, but the sensitivity has been greatly reduced over much of the band. The impedance now is not high as might be expected from a high BL, but it is normal, allowing more

dissipation, showing a nominal peak near the resonance frequency. Thus although the box is small, this approach seems limited in its usefulness. However, there may be other combinations of box size, BI and cone mass which give useful characteristics.



**Figure 15.** Frequency response curves of sealed-box systems with high BI=40N/A and small box of 1 L volume. The thick line is for a normal moving mass of 15 grams, while the thin line is for a heavy cone of 375 grams.

## 10. Summary

When the magnetic field strength of a loudspeaker transducer is significantly increased, the system becomes much more efficient, and the greatly-increased damping causes the effects of the box to be felt only at frequencies far below the normal bass cutoff. Hence the box volume can be very small. The resulting system still needs equalization, but has very appealing characteristics.

If cone mass is increased, the system avoids the need for equalization, but other less attractive features follow.

When a vented system is considered with a driver of high BI, it displays a very under-damped port resonance. This may necessitate real port damping, negating some of the advantages of the high BI.

## 11. Acknowledgements

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