Optimally sensitive and efficient compact loudspeakers

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In conventional loudspeaker system design, the force factor *Bl* is chosen in relation to enclosure volume, cone diameter, and moving mass to yield a flat response over a specified frequency range. For small-cabinet loudspeakers such a design is quite inefficient. This is shown by calculating the efficiency and voltage sensitivity. The frequency response is manipulated electronically in a strong nonlinear fashion, which has consequences for the sound quality, but it then turns out that systems using much lower force factors can provide greater usable efficiency, at least over a limited frequency range. For these low-force-factor loudspeakers, a practically relevant and analytically tractable optimality criterion, involving the loudspeaker parameters, will be defined. This can be especially valuable in designing very compact loudspeaker systems. An experimental example of such a design is described. This new, optimal design has a much higher power efficiency as well as a higher voltage sensitivity than current bass drivers, while the cabinet can be much smaller. [DOI: 10.1121/1.2151694]

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I. INTRODUCTION

There is a longstanding interest in obtaining a high sound output from compact loudspeaker arrangements. Compact relates here to both the volume of the cabinet in which the loudspeaker is mounted as well as to the cone area of the loudspeaker. Loudspeakers can be built such that they properly reproduce the entire audible frequency spectrum, down to 20 Hz; but such systems would be both expensive and very bulky. In many sound reproduction applications it is not possible to use large loudspeaker systems because of size or cost constraints. Typical applications are portable audio, multimedia, and (flat) TV sets. Various signal-processing schemes have been proposed to equalize the response of small loudspeakers or to use psychoacoustic enhancement methods; see Larsen and Aarts (2004) for some overview. The aim of this present paper is to discuss a method to manipulate electronically, in a strong nonlinear fashion, a special loudspeaker with a high acoustical output. The dependence of the transducer's behavior on various parameters, in particular the force factor *Bl*, is investigated. For electrodynamic loudspeakers the perceived quality is important, but also the sensitivity [Pa/V] and the efficiency are of importance. Therefore, in the following section the sensitivity and the efficiency of electrodynamic loudspeakers in general are discussed. It appears that drivers with very high efficiency have poor sensitivity at low frequencies. It is not possible to combine a very high efficiency and a high sensitivity in a wide frequency range with a compact arrangement. In Sec. III special drivers with a very low—but optimal—*Bl* value will be discussed. They have an optimal sensitivity and are only 3 dB less efficient than an infinite-force-factor loudspeaker, but in a limited frequency range only. These characteristics are obtained at the expense of decreased sound quality and the requirement of some additional electronics. Due to the low-*Bl* value, the magnet can be considerably smaller than usual and the loudspeaker can be of the moving-magnet type with a stationary coil, instead of vice versa. In Sec. IV it is discussed how such a low-*Bl* driver can be made. It appears to be very cost-efficient, low-weight, flat, and requires a low-volume cabinet.

II. SENSITIVITY AND EFFICIENCY CALCULATIONS

For low frequencies a loudspeaker can be modeled using some simple elements, allowing the formulation of approximate analytical expressions for the loudspeaker sound radiation (Beranek, 1954; Thiele, 1971; Small, 1972). Neglecting the self-inductance L_e of the driver's voice coil, the transfer function at distance r from voltage $E(s)$ to pressure $P(s)$, also known as the sensitivity, can be written (Aarts, 2005) as

$$H_p(s) = \frac{P(s)}{E(s)} = \frac{s^2 \rho S/(2\pi r)Bl/R_e}{s^2 m_t + sR_t + k_t}, \tag{1}$$

with all used variables as listed in Table I. It appears to be convenient to use the following dimensionless quality factors Q, the dimensionless frequency detuning ν, and resonance frequency ω_0

$$Q_m = \sqrt{k_t m_t}/R_m, \quad Q_e = R_e \sqrt{k_t m_t}/(Bl)^2,$$

$$Q_r = \sqrt{k_t m_t}/R_r, \quad Q_{mr} = Q_m Q_r/(Q_m + Q_r),$$

$$Q_t = (m_t \omega_0)/R_t, \quad \nu = \omega/\omega_0 - \omega_0/\omega, \text{ and } \omega_0 = \sqrt{k_t/m_t}. \tag{2}$$

Using these equations, Eq. (1) can be written as

$$H_p(\omega) = \frac{(i\omega/\omega_0)^2}{(i\omega/\omega_0)^2 + (i\omega/\omega_0)Q_t^{-1} + 1}\left(\frac{\rho a^2 Bl}{2 m_t r R_e}\right). \tag{3}$$

The first fraction on the right-hand side of Eq. (3) expresses the typical high-pass characteristic of a loudspeaker, while the second fraction gives the value for high frequencies ($\omega \gg \omega_0$). At the resonance frequency ($\omega = \omega_0$) Eq. (3) becomes

TABLE I. System parameters of the model.

a	radius of the cone
B	flux density in the air gap
Bl	force factor
E	voice coil voltage
F	$=BlI_c$ is the Lorentz force acting on the voice coil
i	$\sqrt{-1}$
I_c	voice coil current
k_t	total spring constant $=k_d$ (driver alone) $+k_B$ (box)
l	effective length of the voice coil wire
m_t	total moving mass, including the air load mass
ω_0	resonance frequency
$\omega_t=1.4c/a$	transient frequency
P	sound pressure
R_e	electrical resistance of the voice coil
R_m	mechanical damping
R_d	electrodynamic damping $=(Bl)^2/R_e$
R_r	real part of $Z_{\text{rad}}=\Re\{Z_{\text{in}}\}$
R_t	total damping $=R_r+R_m+R_d$
ρ	density of the air
s	Laplace variable
S	surface of the cone with radius a
V	velocity of the voice coil
Z_{rad}	mechanical radiation impedance $=R_r+iX_r$

$$H_p(\omega_0) = \frac{P(\omega_0)}{E(\omega_0)} = \frac{i\rho a^2 Bl\omega_0}{2rR_eR_t}. \tag{4}$$

Equation (4) shows that the sensitivity at the resonance frequency depends on the mass m_t via ω_0. Section IV will elaborate on this, and it is shown that at the resonance frequency it is beneficial to have a low-*Bl* value.

The electrical input impedance can be written (Aarts, 2005) as

$$Z_{\text{in}}(\omega) = R_e\left[1 + \frac{Q_{mr}/Q_e}{1 + iQ_{mr}\nu}\right]. \tag{5}$$

From this it appears that—via Q_e—Bl plays an important role in the electrical impedance, which is most pronounced at the resonance frequency. By neglecting Z_{rad}, which is very small at low frequencies—in particular at ω_0—the electrical input impedance at ω_0 can be approximated as

$$Z_{\text{in}}(\omega_0) \approx R_e + (Bl)^2/R_m. \tag{6}$$

In order to calculate the power efficiency of loudspeakers, it is required to calculate the electrical power delivered to the driver as well as the acoustical power radiated by the loudspeaker. The latter depends on the radiation impedance of the driver. Below, expressions for these three quantities are derived.

Assuming a sinusoidal driving signal, the time-averaged electrical power P_e delivered to the driver can be written as

$$P_e = 0.5|I_c|^2\Re\{Z_{\text{in}}\} = 0.5|I_c|^2R_e\left[1 + \frac{Q_{mr}/Q_e}{1 + Q_{mr}^2\nu^2}\right], \tag{7}$$

where $\Re\{Z_{\text{in}}\}$ is the real (resistive) part of the input impedance Z_{in}. The radiation impedance Z_{rad} of a plane circular rigid piston with a radius a in an infinite baffle can be derived as (Morse and Ingard, 1968, p. 384)

$$Z_{\text{rad}} = \pi a^2\rho c[1 - 2J_1(2ka)/(2ka) + i2\mathbf{H}_1(2ka)/(2ka)], \tag{8}$$

where $\mathbf{H}_1$ is a Struve function and J_1 is a Bessel function (Abramowitz and Stegun, 1972, Secs. 12.1.7 and 9, respectively), and k is the wave number ω/c. An accurate, full-range approximation of $\mathbf{H}_1$ is given in Aarts and Janssen (2003) as

$$\mathbf{H}_1(z) \approx \frac{2}{\pi} - J_0(z) + \left(\frac{16}{\pi} - 5\right)\frac{\sin z}{z} + \left(12 - \frac{36}{\pi}\right)\frac{1-\cos z}{z^2}. \tag{9}$$

For low frequencies ($\omega \ll \omega_t = 1.4c/a$) the damping influence of Z_{rad} can either be neglected, or the following approximation (Aarts, 2005) can be used:

$$R_r = \Re\{Z_{\text{rad}}\} \approx \pi a^2\rho c(ka)^2/2. \tag{10}$$

Assuming $c=343$ m/s, $\rho=1.21$ kg/m^3, Eq. (10) yields

$$R_r \approx (0.15Sf)^2, \tag{11}$$

where $f=\omega/2\pi$.

The time-averaged acoustically radiated power can be calculated as

$$P_a = 0.5|V|^2\Re\{Z_{\text{rad}}\}, \tag{12}$$

which can be written (Aarts, 2005) as

$$P_a = \frac{0.5(Bl/(R_m + R_r))^2 I_c^2 R_r}{1 + Q_{mr}^2\nu^2}. \tag{13}$$

Using Eqs. (7) and (13), the power efficiency can now be calculated as

$$\eta(\nu) = P_a/P_e = [Q_eQ_r(\nu^2 + 1/Q_{mr}^2) + Q_r/Q_{mr}]^{-1}. \tag{14}$$

This function depends on all loudspeaker parameters and the frequency. In classical loudspeaker design theory the parameters are chosen such that the sensitivity function $H_p(\omega)$ given by Eq. (3) has a flat characteristic for $\omega > \omega_0$, which implies that $Q_t \approx 1/\sqrt{2}$. This gives little freedom in the design parameters. Furthermore, one wants a reasonable efficiency. Recently, Vanderkooy *et al.* (2003) investigated the use of high-*Bl* drivers. The aim of that study was to obtain efficient loudspeakers; however, they have a poor sensitivity at low frequencies. In the following section the use of low-*Bl* drivers is discussed; those drivers appeared to be highly sensitive and exhibit a good efficiency, but only around the resonance frequency.

III. SPECIAL DRIVERS FOR LOW FREQUENCIES

Two options are described whereby modifying a conventional loudspeaker driver can lead to enhanced bass performance. This is achieved by modifying the force factor of the driver, in particular by employing either a very strong or very weak magnet compared to what is commonly used in typical drivers. Both these approaches also require some preprocessing of the signal before it is applied to the modified loud-

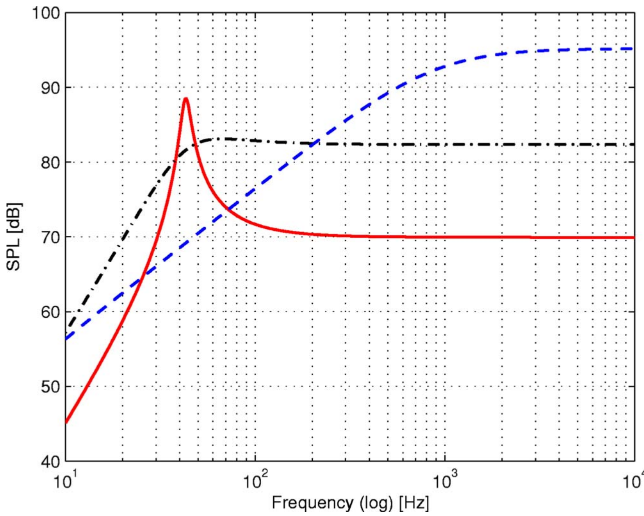

FIG. 1. (Color online) Sound-pressure level (SPL) for the driver MM3c with three Bl values: low $Bl=1.2$ (solid), medium $Bl=5$ (dash-dot), and high $Bl=22$ N/A (dash), while all other parameters are kept the same as given in Table II, all with 1-W input power and at 1 m distance. At the resonance frequency, the highest SPL is obtained by the low-Bl driver, while the high-Bl driver has at low frequencies—in particular at the resonance frequency—a poor response.

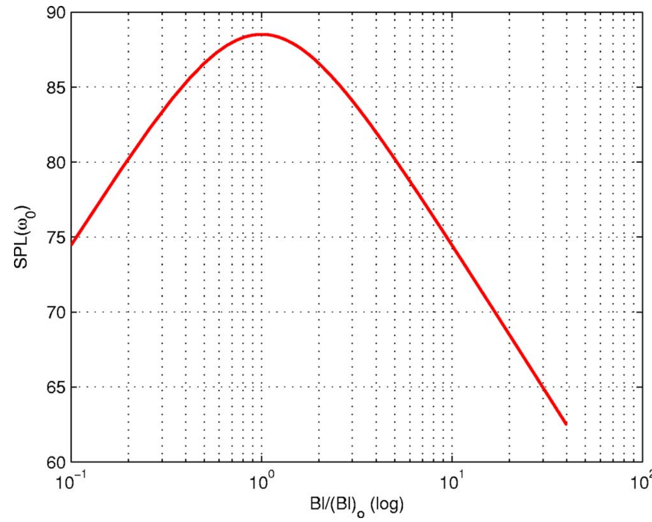

FIG. 2. (Color online) The SPL at the resonance frequency versus the normalized force factor $Bl/(Bl)_o$ for the driver MM3c, where $(Bl)_o$ is the optimal force factor given in Eq. (29), in this present case $(Bl)_o=1.19$. The other parameters are given in Table II.

speaker. In the remaining section the influence of the force factor on the performance of the loudspeaker is reviewed.

Direct-radiator loudspeakers typically have a very low efficiency, because the acoustic load on the diaphragm or cone is relatively low compared to the mechanical load. In addition, the driving mechanism of a voice coil is quite inefficient in converting electrical energy into mechanical motion. The force factor Bl is deliberately kept at an intermediate level so that the typical response is sufficiently flat to use the device without significant equalization. It was already shown in Sec. II that the force factor Bl plays an important role in loudspeaker design. It determines among others the frequency response and its related transient response, the electrical input impedance, and the weight of a loudspeaker; the following will discuss these various characteristics.

To show the influence on the frequency response, the sound-pressure level (SPL) of a driver with three Bl values (low, medium, and high) is plotted in Fig. 1, while all other parameters are kept the same.

It is seen that the curves change drastically for varying Bl. The most prominent difference is the shape, but also apparent is the difference in level at high frequencies. While the low-Bl driver has the highest response at the resonance frequency, it has a poor response beyond resonance, so in use this loudspeaker requires special treatment, as discussed in Sec. IV. The high-Bl driver has a good response at higher frequencies, but a poor response at lower frequencies, which requires special equalization. In between, there is the well-known curve for a medium-Bl driver. The influence of Bl on the sensitivity at the resonance frequency is further clarified by plotting the SPL at the resonance frequency versus the normalized Bl, as is shown in Fig. 2.

It appears that at the resonance frequency there is an optimal value for the voltage sensitivity at $Bl/Bl_o=1$, where Bl_o is the optimal-Bl value discussed in Sec. IV. The underlying reason for the importance of Bl is that, besides determining the driving force, it also provides (electrodynamic) damping to the system. The total damping R_t is equal to the sum of the real part of the radiation impedance R_r, the mechanical damping R_m, and the electrodynamic damping $R_d=(Bl)^2/R_e$, where the electrodynamic damping dominates for medium- and high-Bl loudspeakers, and is most prominent around the resonance frequency. The variables in this electrodynamic damping term cannot be selected independently. This can be seen as follows. The voice coil resistance can be written as

$$R_e = \frac{l\rho_e}{A_e}, \tag{15}$$

where ρ_e and A_e are the electric conductivity and area of the voice coil wire, respectively. The volume occupied by the voice coil is equal to

$$V_e = A_e l. \tag{16}$$

Combining these two equations yields the electrodynamic damping

$$R_d = \frac{(Bl)^2}{R_e} = \frac{B^2 V_e}{\rho_e}, \tag{17}$$

which shows that the volume occupied by the voice coil, and the material used for the magnet and voice coil wire, determines the electrodynamic damping, and not the length l of the voice coil's wire.

The power efficiency given in Eq. (14) can be written as

$$\eta(\omega) = \frac{(Bl)^2 R_r}{R_e\{(R_m+R_r)^2 + (R_m+R_r)(Bl)^2/R_e + (m_t\omega_0\nu)^2\}}. \tag{18}$$

If $(m_t\omega_0\nu)^2 \gg [(R_m+R_r)^2+(R_m+R_r)(Bl)^2/R_e]$, and R_r is approximated by Eq. (10), then Eq. (18) can be written as

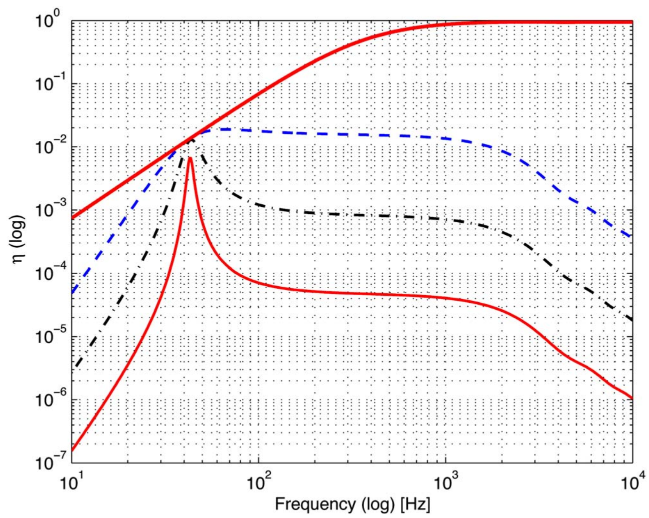

FIG. 3. (Color online) The power efficiency η for the driver MM3c with four Bl values: low $Bl=1.2$ (solid), medium $Bl=5$ (dash-dot), high $Bl=22$ N/A (dash), and $\lim_{Bl\to\infty}$ (thick solid), while all other parameters are kept the same as given in Table II. Note that the efficiency is strongly dependent on Bl at all frequencies except at resonance, where the efficiency is affected only modestly by Bl.

$$\eta(\omega_0 \ll \omega \ll \omega_t) \approx \frac{(Bl)^2 S^2 \rho}{2\pi c R_e m_t^2}. \tag{19}$$

This is a well-known result in the literature (Beranek, 1954) and clearly shows the influence of Bl, however, is valid in a limited frequency range only.

Using Eq. (18), the power efficiency η is plotted in Fig. 3, which clearly shows the dependency on frequency.

Figure 3 shows the efficiency function η as function of the frequency for various values of Bl, while all other parameters are kept the same. It appears that the curves change drastically for varying Bl, but only very modestly around the resonance frequency. This can further be clarified by using Eq. (18) and calculating the limit

$$\lim_{Bl\to\infty} \eta(\omega) = \frac{R_r(\omega)}{R_m + R_r}. \tag{20}$$

The curve for $\eta(\omega)$ for this infinite-Bl value is the thick-solid curve in Fig. 3. Assuming that $R_r \ll R_m$ and $\omega=\omega_0$, and using Eqs. (10) and (20), this yields at the resonance frequency

$$\lim_{Bl\to\infty} \eta(\omega=\omega_0) \approx \frac{R_r(\omega_0)}{R_m} \approx \frac{\rho(S\omega_0)^2}{2\pi c R_m}. \tag{21}$$

Equation (21) shows the approximate value of the power efficiency at the resonance frequency for infinite Bl. It appears that the four curves of Fig. 3 are almost coincident at the point given by Eq. (21), even for the low-Bl curve. This is further elucidated in Fig. 4. This graph shows the power efficiency at the resonance frequency versus Bl/Bl_o, where Bl_o is the optimal-Bl value discussed in Sec. IV.

Figure 4 shows an s curve, where the part for very low-Bl values exhibits a very poor efficiency. There, the Lorentz force acting on the driver's voice coil is small with respect to the damping. Then, a rather steep part of the curve follows, and finally a plateau exists, which is given by Eq. (20). The importance of Bl is further elucidated in the following section.

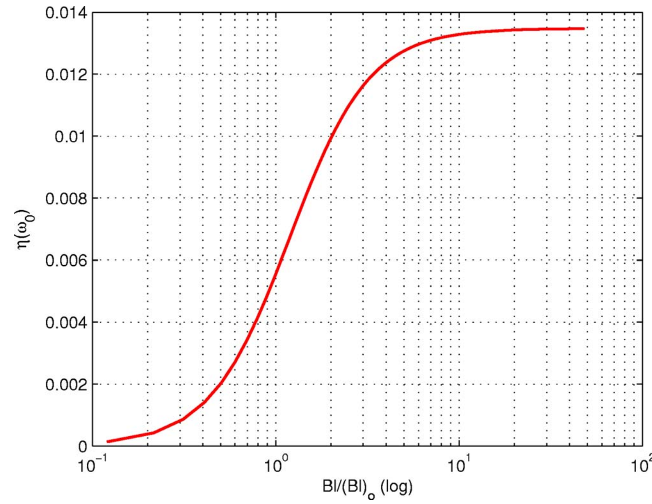

FIG. 4. (Color online) The power efficiency $\eta(\omega=\omega_0)$ versus the normalized force factor $Bl/(Bl)_o$ for the driver MM3c, where $(Bl)_o$ is the optimal force factor given in Eq. (29), in this present case $(Bl)_o=1.19$. The other parameters are given in Table II.

IV. LOW-FORCE-FACTOR DRIVERS

As explained before, normally low-frequency sound reproduction with small transducers is quite inefficient. Two measures are proposed to increase the efficiency. First, a special transducer is used with a low-Bl value, attaining a high efficiency and the highest possible sensitivity at that particular frequency. Second, nonlinear processing essentially compresses the bandwidth of a 20- to 120-Hz bass signal down to a much narrower span. This span is centered at the resonance of the low-Bl driver where its efficiency is maximum. These drivers are only useful for subwoofers. In the following an optimal force factor is derived to obtain such a result.

The proposed solution, to obtain a high sound output from a compact loudspeaker arrangement with a good efficiency, consists of two steps. First, the requirement that the frequency response must be flat is relaxed. By making the magnet considerably smaller and lighter (see Fig. 5, right

FIG. 5. (Color online) Picture of the prototype driver (MM3c) with a 10 Euro cents coin. At the position where a normal loudspeaker has its heavy and expensive magnet, the prototype driver has an almost empty cavity; only a small moving magnet is necessary, which is shown in the right corner.

TABLE II. The lumped parameters of the new, and experimental driver with the (optimal) low-*Bl* MM3c; see Fig. 5 for its compact magnet system. See Table I for the abbreviations and the meaning of the variables.

Type	MM3c
R_e Ω	6.4
Bl N/A	1.2
k_d N/m	1022
m_t g	14.0
R_m Ns/m	0.22
S cm^2	86
f_0 Hz	43
Q_m	17.2
Q_e	16.8

side) a large peak in the SPL curve [see Fig. 1 (solid curve)] will appear.

Because the magnet can be considerably smaller than usual, the loudspeaker can be of the moving magnet type with a stationary coil (see Fig. 5) instead of vice versa. At the resonance frequency the voltage sensitivity can be a factor of 10 higher than that of a normal loudspeaker. In this case an SPL of almost 90 dB at 1-W input power at 1-m distance is achieved at the resonance frequency, even when using a small cabinet (<1 1). Because it is operating in resonance mode only, the moving mass can be enlarged—which might be necessary owing to the small cabinet—to keep the resonance frequency sufficiently low. This is done without degrading the efficiency of the system because at the resonance frequency $\nu=0$ and the product $m_t\omega_0\nu$ in Eq. (18) becomes equal to zero. See Table II.

Due to the high and narrow peak in the frequency response, the normal operating range of the driver decreases considerably. This makes the driver unsuitable for normal use. To overcome this, a second measure is applied. Nonlinear processing essentially compresses the bandwidth of a 20- to 120-Hz 2.5-octave bass signal down to a much narrower span—which is centered at the resonance of the low-*Bl* driver—where its efficiency is maximum. This can be done with a setup as depicted in Fig. 6 and will be discussed below.

Without loss of generality, it is assumed that the low-frequency part of the music can be modeled as a sinusoid with frequency ω_c which is modulated by a slowly varying signal $m(t) \geqslant 0$. This yields

$$y(t) = [c_m + m(t)]\sin(w_c t), \tag{22}$$

where c_m is a constant, or more precisely

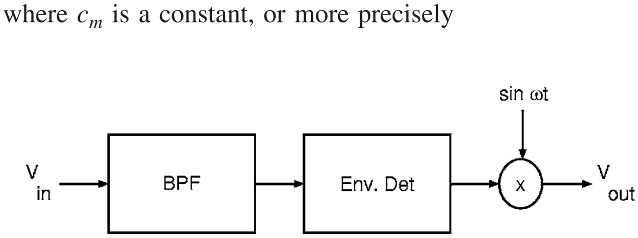

FIG. 6. Frequency mapping scheme. The box labeled "BPF" is a bandpass filter, and "Env. Det." is an envelope detector. The latter can be a simple rectifier followed by a low-pass filter. The signal V_{out} is fed (via a power amplifier) to the driver.

$$h = \frac{\text{peak value of } m(t)}{c_m} \tag{23}$$

is the modulation index. This model is realistic since $y(t)$ is a bandlimited signal, say between 20 to 120 Hz. The frequency of ω_c can be variable and will lead to a certain pitch. Taking the Fourier transform of Eq. (22), the magnitude of the spectrum can be written as

$$|Y(\omega)| = \pi c_m \delta(\omega - \omega_c) + \tfrac{1}{2}M(\omega - \omega_c) + \pi c_m \delta(\omega + \omega_c) + \tfrac{1}{2}M(\omega + \omega_c), \tag{24}$$

where δ is a unit impulse and the capital function $M(\omega)$ indicates the Fourier transform of $m(t)$. Equation (24) shows the well-known amplitude-modulated (AM) spectrum, as known from AM radio broadcasting. In contrast to normal AM radio, in the present case $c_m=0$, this is to make the amplitude of $y(t)$ proportional to the amplitude of $m(t)$. If the processing depicted in Fig. 6 is applied to the signal $y(t)$, the signal $m(t)$ is recovered by an envelope detector and is used to modulate a sinusoid, but now with frequency ω_0, where ω_0 is fixed and equal to the resonance frequency of the transducer. This yields

$$v_{\text{out}}(t) = m(t)\sin(w_0 t), \tag{25}$$

with the corresponding spectrum

$$|V(\omega)| = \tfrac{1}{2}[M(\omega - \omega_0) + M(\omega + \omega_0)]. \tag{26}$$

The result is that the coarse structure $m(t)$ (the envelope) of the music signal after the compression or "mapping" is the same as before the mapping; an example is shown in Fig. 7. Only the fine structure has been changed to a sinusoid of the same frequency as the driver's resonance frequency.

The upper panel shows the waveform of a rock-music excerpt; the thin curve depicts its envelope, $m(t)$. The middle and lower panels show the spectrograms of the input and output signals, respectively, clearly showing that the frequency bandwidth of the signal around 60 Hz decreases after the mapping, yet the temporal modulations remain the same.

Using Eq. (1) and neglecting Z_{rad}, the voltage sensitivity at the resonance frequency can be written as

$$H_p(\omega_0) = \frac{P(\omega_0)}{E(\omega_0)} = \frac{i\omega_0 SBl\rho}{2\pi r R_e(R_m + (Bl)^2/R_e)}. \tag{27}$$

Equation (27) is maximized by adjusting the force factor Bl by differentiating $H_p(\omega=\omega_0)$ with respect to Bl and setting $\partial H_p/\partial(Bl)=0$, resulting in

$$\frac{(Bl)^2}{R_e} = R_m. \tag{28}$$

It appears that the maximum voltage sensitivity is reached when the electrodynamic damping term $(Bl)^2/R_e$ is equal to the mechanical damping term R_m; in this case the optimal force factor is defined as

$$(Bl)_o = \sqrt{R_e R_m}. \tag{29}$$

The consequences of this optimality criterion are discussed below. One obvious observation is that the SPL response becomes, as can be seen in Fig. 1 (solid curve), very peaky.

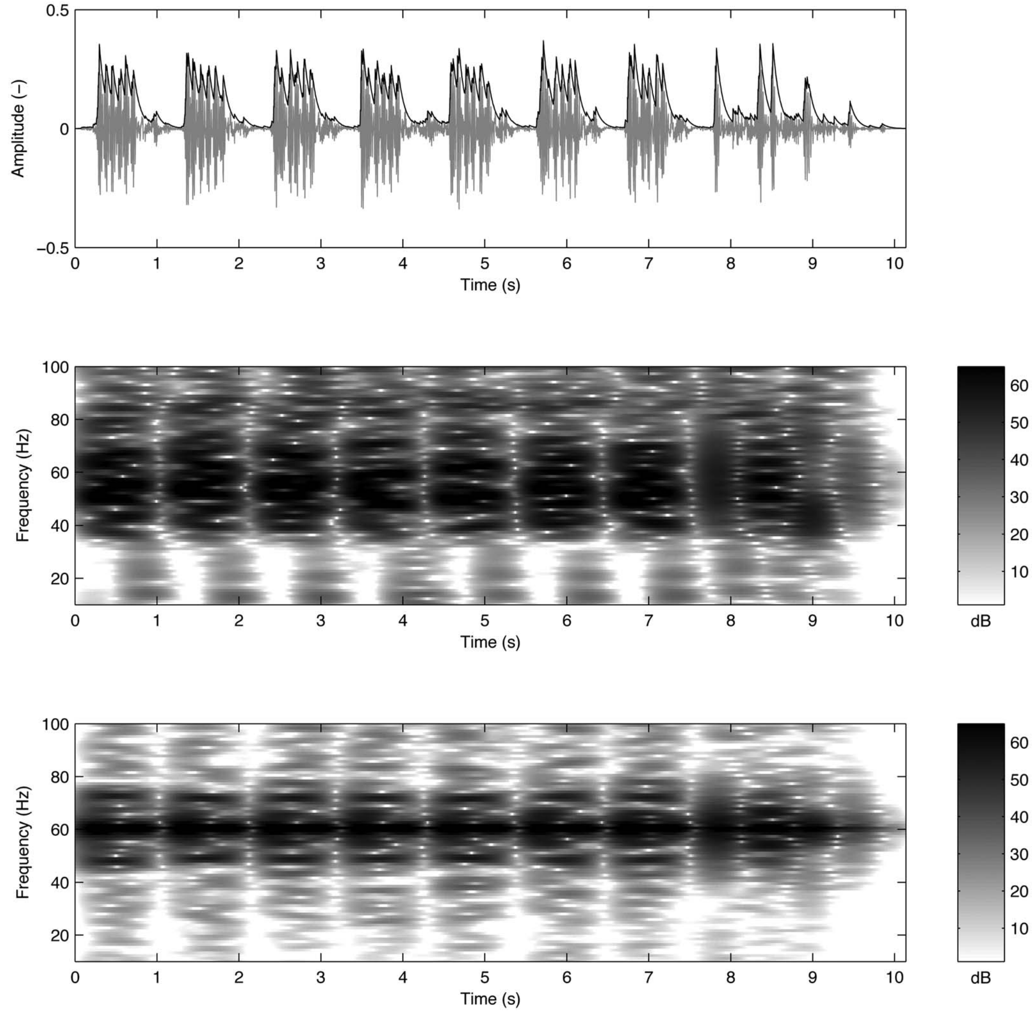

FIG. 7. The signals before and after the frequency-mapping processing of Fig. 6. The upper panel shows the time signal at V_{in}, and the thin curve the output of the envelope detector. The middle and lower panels show the spectrogram of the input and output signals, respectively.

The height of the peak is calculated by substituting Eq. (28) into Eq. (27), which yields the optimal voltage sensitivity

$$H_o(\omega = \omega_0) = \frac{i\omega_0 \rho S}{4\pi r (Bl)_o}. \quad (30)$$

The specific relationship between $(Bl)_o$ and both R_m and R_e [Eq. (29)] causes H_o to be inversely proportional to $(Bl)_o$ (which may seem counterintuitive), and thus also inversely proportional to $\sqrt{R_m}$. For this particular value of Bl the Lorentz force is large enough to get a sufficiently strong driver with good efficiency, but the electromagnetic damping is sufficiently low to reach the optimal sensitivity.

The power efficiency at the resonance frequency under the optimality condition, obtained by substitution of Eq. (28) into Eq. (18), yields

$$\eta_o(\omega = \omega_0) = \frac{R_m R_r}{(R_m + R_r)^2 + (R_m + R_r) R_m}. \quad (31)$$

This can be approximated for $R_r \ll R_m$ as

$$\eta_o(\omega = \omega_0) \approx \frac{R_r}{2R_m}, \quad (32)$$

which clearly shows that for a high power efficiency at the resonance frequency, the cone area must be large, because R_r—according to Eq. (8), or more explicitly Eq. (11)—is proportional to the squared cone area, and that the mechanical damping must be as small as possible. The damping must be not too small, however, because the transient response depends on the damping as well, as is discussed in Larsen and Aarts (2004). Comparing Eq. (32) with Eq. (21) shows that the optimally sensitive driver is only 3 dB less efficient than the infinite Bl one; however, this is only at the resonance frequency, but this is the working frequency of the new driver. This can also be seen in Fig. 3 where η_0 (solid curve) is close to the infinite Bl curve, but only at the resonance frequency.

V. DISCUSSION

Sound reproduction at low frequencies with small drivers in small cabinets is not efficient. Small drivers have a low radiation impedance with respect to the total damping [see Eq. (18)]. Small cabinets have a stiff air spring which needs a high moving mass to obtain the desired low resonance frequency. This will be reiterated more quantitatively below. For a given volume of the enclosure V_0, the corresponding k_B of the "air spring" can be calculated as

$$k_B = \frac{\rho(cS)^2}{V_0}. \tag{33}$$

Mounting a loudspeaker in a cabinet will increase the total spring constant k_t by an amount given by Eq. (33), and subsequently increase the resonance frequency of the system. To compensate for this bass loss, the moving mass has to be increased; thus, $\sqrt{k_t m_t}$ is increased, which raises Q_e [see Eq. (2)]. Then—to obtain a flat frequency characteristic—*Bl* must be increased to preserve the original value of Q_e. This is at the cost of a more expensive magnet and a loss in efficiency. This is the designer's dilemma: high efficiency or small enclosure? To meet the demand for a certain cutoff frequency, the enclosure volume must be larger. Alternatively, the efficiency for a given volume will be less than for a system with a higher cutoff frequency.

This dilemma is (partially) solved by using the low-*Bl* concept as discussed in Sec. IV, however, at the expense of a decreased sound quality and the need for some additional electronics to accomplish the frequency mapping. While the new driver is not a hi-fi one, many informal listening tests and demonstrations[1] confirmed that the decrease of sound quality appears to be modest, apparently because the auditory system is less sensitive at low frequencies. Also, the other parts of the audio spectrum have a distracting influence on this mapping effect, which has been confirmed during formal listening tests (Le Goff *et al.*, 2004), where the detectability of mistuned fundamental frequencies was determined for a variety of realistic complex signals. Finally, the part of the spectrum which is affected is only between, say, 20 and 120 Hz, so the higher harmonics of these low notes are mostly out of this band and are thus not affected. They will contribute in their normal unprocessed fashion to the missing fundamental effect. All these factors support the notion that detuning becomes difficult to detect once the target complex is embedded in a spectrally and temporally *rich* sound context, as it is typical for applications in modern multimedia reproduction devices (Le Goff *et al.*, 2004).

VI. CONCLUSIONS

The force factor *Bl* plays a very important role in loudspeaker design. It determines the efficiency, the sensitivity, the impedance, the SPL response, the weight, and the cost. It appears to be not possible to obtain both a high efficiency as well as a high sensitivity in a wide frequency range. At the loudspeaker's resonance frequency, however, it appears to be possible to meet this criterion. The voltage sensitivity is optimal when the electrical damping force is equal to the mechanical one, while it is only 3 dB less efficient than an infinite force-factor loudspeaker. A new low-*Bl* driver has been developed which together with some additional electronics, yields a low-cost, lightweight, compact, physically flat, optimally sensitive, and very-high-efficiency loudspeaker system for low-frequency sound reproduction.

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[1]Demonstrations and MATLAB scripts are on http://www.dse.nl/~rmaarts

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