

A simple trigonometric function with some interesting properties

The function

$$y(x) = \frac{\sin(x)}{2 + \cos(\alpha x)}, \quad (1)$$

has some interesting properties, depending on its parameter α .

For $\alpha = 0$, $y(x)$ reduces to a simple sinusoid.

For $\alpha = 1/2$, the period of $y(x)$ doubles to 4π .

For $\alpha = 1$, the curve of $y(x)$ is sawtooth like. The even order coefficients of its Taylor series at 0 and π are equal to zero, but the third order at 0 is equal to zero as well. Therefore the slope at 0 is rather linear. The Fourier series has sine terms only, the coefficients of the odd terms are positive, those of the even ones are negative.

For $\alpha = \sqrt{2}$, or any other real irrational number, $y(x)$ is an almost periodic function.

For $\alpha = 2$ the curve of $y(x)$ is triangle like. The even order coefficients of its Taylor series at 0 and π are equal to zero. The Fourier series has sine terms only, the coefficients of the odd terms are positive and alternating in sign, the coefficients of the even ones are equal to zero.

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