A non-periodic tiling with a single convex polygon

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A tiling is a covering of the plane by non-overlapping polygons or other shapes without gaps, see [3] for some history and recent results. An aperiodic means that shifting any tiling with these shapes by any finite distance, without rotation, cannot produce the same tiling [2, Ch 10]. A Penrose tiling is an example of an aperiodic tiling [2, Sec.10.3]. There are various versions, one is a pair of rhombuses (often referred to as rhombs in this context, or P3) with equal sides but different angles [2, p.542]. Ordinary rhombus-shaped tiles can be used to tile the plane periodically, also the Penrose rhombs can tile the plane periodically. Therefore, restrictions must be made on how tiles can be assembled. Penrose devised matching rules e.g. with colors to obtain an aperiodic tiling [2, p.542].

Here, a tiling is proposed which is using a regular hexagon cut through the center of the hexagon into two congruent pentagons resulting in a specific type 1 [2, p.492] pentagon. Likewise the Penrose's rhombs, type 1 pentagons can tile the plane periodically, but to prevent this, matching rules are applied too. The first rule is that the pair of pentagons are positioned such that it remains a regular hexagon. A potential second rule is to mark the edges of the hexagon like the P3 to obtain an aperiodic tiling, but it is yet unknown whether this is possible. It was shown [1] that the properties of Penrose tilings are reminiscent of the quasi-periodic behavior of pseudo random sequences of arbitrary length, known as de Bruijn sequences. Therefore, as the second rule, the proposed divided hexagons are rotated directly by a (pseudo)random selection of 0, $\pi/3$, $2\pi/3$. Resulting in a non-periodic tiling with one convex pentagon as the protile. Variations on this theme are obvious, e.g. by cutting the hexagon into two congruent quadrilaterals, or three congruent pentagons and then rotating as discussed.

REFERENCES

1. de Bruijn, N.G., (1981). Algebraic theory of Penrose's non-periodic tilings of the plane, *Kon. Nederl. Akad. Wetensch. Proc. Ser. A.***84**: 39–66. doi.org/10.1016/1385-7258(81)90016-0
2. Grünbaum, B., Shephard, G.C., (2016). *Tilings and Patterns*, 2nd ed. Garden City, NY: Dover.
3. Zong, C., (2020). Can You Pave the Plane with Identical Tiles?, *Notices AMS.* **67**(5): 635–646. doi.org/10.1090/noti2075