Simulation of Loudspeaker Crossover Filters with a Digital Signal Processor*

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A method is presented for the evaluation of crossover filters in a multiway loudspeaker system. A number of analog filters, designed by "hand" or with CAD tools, are potentials for use in a loudspeaker system. These different filters are real-time simulated by a digital audio signal processor, which permits flexible and accurate switching between different crossover filters as needed in a listening test. The transformation of the analog filter to its digital equivalent and the implementation in a digital signal processor are discussed.

0 INTRODUCTION

Using a software package previously made for the numerical optimization of a multiway loudspeaker system crossover filter [1], a set of component values is given for the crossover filter that satisfies the designer's demands and constraints to the greatest extent possible. In practice, however, many sets are found having different filter topologies or different initial values, for example, because of different specifications of the optimization targets. These various sets can be differentiated by examining their numerical qualities (such as standard deviation) or by comparing their sensitivities to component value tolerances. It is our experience that after such a differentiation, several almost optimum solutions (sets of component values) remain, and that such solutions with almost identical "numerical qualities" can sound quite different. Therefore we need a "final check" to select the best-sounding solution. Obviously, such a final check is a listening test. In such a listening test it is desirable for the designer to switch between different crossover filter candidates, without changing the loudspeakers or the location of the loudspeaker system. Meticulous attention to the acoustical, psychological, and experimental variables is required to achieve subjective ratings that are reliable [2]. This can be achieved if a "device" is available that emulates the crossover filter, that is, it performs the same task as the analog filter. We will discuss passive crossover filters only, but the same techniques can be used for active systems. For a good comparison between the candidates it is essential to do the listening test very accurately. The above-mentioned reasons justify, in our opinion, the need for an accurate and flexible crossover switching device. This device enables the crossover filters to be changed without side effects, that is, the same loudspeakers are used and the location of system and listener remains unchanged. This device can be built with analog crossover filters and a high-quality switching facility. However, since this approach is quite cumbersome, we have implemented such a device with a digital audio signal processor (ASP) [3].

Filtering digitally has some definite advantages.

1) It provides great flexibility.

2) It can switch fast and accurately between many different crossover filters.

3) All functions can be software controlled.

4) It can be conveniently coupled with a computer-driven listening test.

A disadvantage is the need for rather complex hardware and software tools, which are described below.

1 SIMULATION OF A CROSSOVER FILTER

The simulation of a crossover filter by means of a digital signal processor means that it provides complex-valued transfer functions equal to those between the electric input of the crossover filter and the loudspeaker

* Manuscript received 1987 January 15; presented at the 82nd Convention of the Audio Engineering Society, London, 1987 March 10–13.

terminals, that is, the transfer function of the filter loaded with the complex-valued input impedance of the loudspeaker (Fig. 1). The digital filter is implemented with an ASP. To realize the digital filters we need a transformation from the analog domain (the s or $j\omega$ plane) to the digital one (the z plane), where s is introduced by means of a Laplace transformation and z by means of the z transformation [4]. When the "analog" transfer function is given analytically, standard techniques are available for mapping from the analog to the digital one. In the case in question these cannot be used because an analytical description of the transfer function is not available. This transfer function is given at discrete frequency points only, because the input impedance of the loudspeaker has been determined by measurement at a finite number of frequencies. This necessitates the use of optimization or curve-fitting techniques for mapping the analog filter to its digital equivalent. The transfer function of the digital filter is written as the ratio of two z polynomials [4], and we have to find the coefficients of these polynomials to simulate the specified complex-valued transfer function as accurately as possible.

1.1 Selecting the Type of Digital Filter

The choice to be made is whether the denominator of the "digital" transfer function of $H(z)$ equals unity [a finite impulse response (FIR) filter] or is given by a polynomial [an infinite impulse response (IIR) filter]. Despite the advantages of an FIR filter, its unconditional stability, the relatively easy design, and its lack of limit cycles, we opted for an IIR approach. We did so because an appropriate FIR filter would have to be rather long in order to match the desired accuracy at low frequencies. A disadvantage of the IIR filter approach is the difficulty of determining the coefficients and the orders of the polynomials. When determining the coefficients of the polynomials we have to account for

1) A correction for the spectral shaping of the digital-to-analog converter (DAC) [4]

2) The influence of the finite word length of the coefficients (that is, the number of bits used for each coefficient)

3) The calculated filter being stable, which requires that the roots of the denominator polynomial be located inside the unit circle in the (complex) z plane.

The next section describes how the order of the polynomials and the initial values for the polynomial coefficients which serve as an input for an optimization procedure are estimated. These initial values are very important for a successful optimization, which determines the coefficients for the filter polynomials.

2 NUMERICAL ESTIMATION OF THE DIGITAL FILTER

The desired transfer function is not given analytically but known at discrete frequency points only. The value of the z-domain transfer function depends in a nonlinear way on its (unknown) polynomial coefficients. This makes it hard to determine the coefficients of the z-domain polynomials directly. Consequently we make use of a numerical optimization technique. Such an optimization requires an initial value estimate. The choice of the initial values can be critical because the problem is nonlinear [5]. For a nonlinear optimization, convergence to a solution depends on the initial values of the coefficients, and it is therefore very important to supply good initial values for the coefficients.

In the following we derive an estimate for the analytical crossover transfer function, which is used to estimate the orders of the z-domain polynomials and an initial value for their coefficients.

2.1 Calculation of an Initial Value for the z-Domain Polynomials

The crossover filter optimization program [1] yields the transfer function of the filter (loaded with the loudspeaker) at discrete frequency points and a set of filter

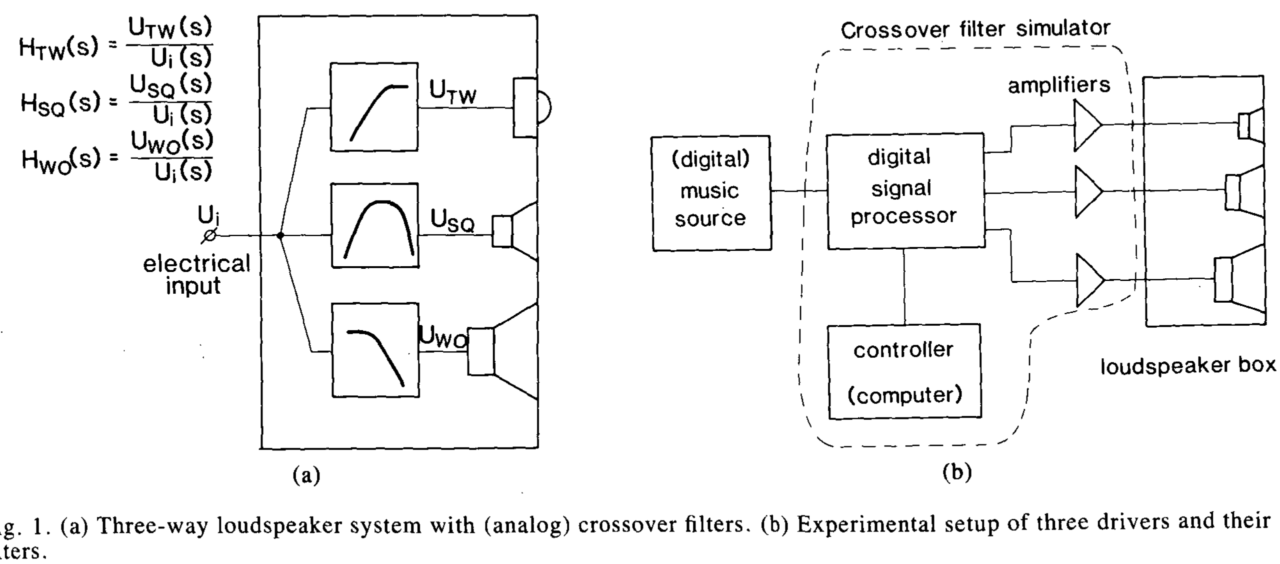

Fig. 1. (a) Three-way loudspeaker system with (analog) crossover filters. (b) Experimental setup of three drivers and their filters.

component values. This information, together with the filter topology, is used to determine an analytical s-domain estimate of the transfer function. This is used to find an initial value for the z-domain transfer function.

The loudspeaker input impedance greatly influences the transfer function of the analog filter and cannot be ignored. An analytical approximation of this input impedance can be derived from the lumped-parameter circuit of the loudspeaker [6] and yields

$$Z_{\mathrm{i}} = R_{\mathrm{E}} + \mathrm{j}\omega L_{\mathrm{E}} + \frac{R_{\mathrm{m_e}}}{1 + \mathrm{j}Q_{\mathrm{m}}(\omega/\omega_0 - \omega_0/\omega)} \quad (1)$$

where R_{E} denotes the dc resistance of the voice coil, L_{E} its self-inductance, $R_{\mathrm{m_e}}$ is the mechanical resistance transferred to electrical quantities, Q_{m} is the mechanical quality factor, ω_0 is the resonance frequency of the loudspeaker, and ω is the angular frequency.

The lumped-parameter values can be estimated roughly from the knowledge of whether the loudspeaker is a low-, mid-, or high-frequency loudspeaker, or, preferably, they can be estimated more precisely from a measured input impedance [7]. The topology and the component values of the crossover filter are known, and we can calculate the transfer function loaded with the loudspeaker in analytical form (see Appendix). This analytical representation for the s-domain transfer function is transformed to the z-domain using the bilinear transform, that is, the complex frequency variable s is replaced by

$$s = 2f_{\mathrm{s}} \frac{(1 - z^{-1})}{(1 + z^{-1})} \quad (2)$$

where f_{s} denotes the sampling frequency.

The value of a high-order s- or z-domain polynomial is in general very sensitive to a truncation of its coefficients. Also the application of the bilinear transformation to such a polynomial yields large expressions which are not easy to handle. Therefore the polynomials are split into a cascade of first- and second-order sections, which reduce the sensitivity to coefficient truncation considerably. The splitting into lower order sections is performed as follows. The poles and zeros of the s-domain transfer function are calculated. These poles and zeros are either real-valued or complex-conjugate pairs. A real pole and a real zero can be combined to form a first-order section

$$\frac{s - n_1}{s - p_1} \quad (3)$$

where n_1 denotes the zero and p_1, the pole. Application of the bilinear transform (2) to expression (3) yields

$$\frac{(2f_{\mathrm{s}} - n_1) - (2f_{\mathrm{s}} + n_1)z^{-1}}{(2f_{\mathrm{s}} - p_1) - (2f_{\mathrm{s}} + p_1)z^{-1}} \,. \quad (4)$$

Complex-conjugate or real-valued poles and zeros can be combined to form a second-order section,

$$\frac{(s - n_2)(s - n_3)}{(s - p_2)(s - p_3)} \quad (5)$$

and application of the bilinear transform yields the second-order z-domain section

$$\frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}} \,. \quad (6)$$

The parameters are given by

$$a_0 = 4f_{\mathrm{s}}^2 - (n_2 + n_3)2f_{\mathrm{s}} + n_2 n_3$$

$$a_1 = - 8f_{\mathrm{s}}^2 + 2n_2 n_3$$

$$a_2 = 4f_{\mathrm{s}}^2 + (n_2 + n_3)2f_{\mathrm{s}} + n_2 n_3$$

$$b_0 = 4f_{\mathrm{s}}^2 - (p_2 + p_3)2f_{\mathrm{s}} + p_2 p_3$$

$$b_1 = - 8f_{\mathrm{s}}^2 + 2p_2 p_3$$

$$b_2 = 4f_{\mathrm{s}}^2 + (p_2 + p_3)2f_{\mathrm{s}} + p_2 p_3 \,.$$

When a (complex) pole and zero are very close in the z plane and their joint influence is of only minor importance in the total transfer function, that is, a deviation with respect to the "analog" transfer function within one half of a decibel, they are both removed. This reduces the order of the polynomials. The product of the transfer functions of all remaining first- and second-order sections is the z-domain function $H(z)$, which serves as an initial value for the optimization. Having found estimates for both the order and the coefficients of $H(z)$, we can proceed with the numerical optimization.

2.2 Numerical Optimization of the z-Domain Polynomials

In the previous section we established the order of the z-domain polynomials and the initial value for their coefficients. These are required for the numerical optimization in order to obtain a good set of coefficients, that is, a set that ensures the complex values of the z-domain function $H(z)$ to approach those of the specified function values as closely as possible.

The optimization procedure searches for coefficients in order to minimize the difference, expressed as a least squares sum, between the specified target function (the analog filter response) and the function to be optimized. The kernel of the actual optimization program is an NAG library [8] routine. It searches for an unconstrained minimum of a least squares sum, which serves as a measure of fit for a nonlinear optimization. It is important both to provide a reasonable set of initial coefficient values and to match the optimization process to the crossover simulation problem. It is our experience that this greatly improves the numerical optimization. The specific measures are listed below.

1) The transfer function $H(z)$ is expressed as a cascade

of second-order sections (each is the ratio of two second-order polynomials) instead of a ratio of two high-order polynomials. In this case the function value of $H(z)$ is much less sensitive to a coefficient truncation, which greatly improves the optimization process. This technique is also important in a practical implementation of the filter when the coefficients have to be rounded to a finite wordlength, but even with the much higher wordlength of a mainframe computer this is important, too.

2) The frequency points used in the calculation are located equidistantly on a logarithmic frequency scale.

3) The terms in the least squares sum, the objective function, are weighted with a frequency-dependent weighting function in order to weaken the influence of points whose amplitude is much lower than in the passband. The weighting function equals the magnitude of the specified "analog" transfer function expressed in decibels.

4) All sections are normalized with respect to gain, and an overall gain factor or attenuation is defined, which reduces the number of variables to be optimized.

5) The z-domain function $H(z)$ is multiplied by a time-delay function. The set of optimization variables is extended with the delay. This is done to deal with possible differences in delay between the analog and the digital filter.

6) An unstable section is rejected by applying a penalty factor to the objective function, that is, the least squares sum is multiplied by a large number.

3 IMPLEMENTATION

Numbers in a digital signal processor are represented by binary words with a limited number of bits. As a consequence, errors are introduced which can seriously affect a practical implementation.

A first error to be mentioned is the rounding of the coefficients from mainframe computer precision to signal processor precision. The effect is that the transfer functions before and after rounding can differ considerably; in the worst case the filter may even become unstable.

A second error source is the finite precision of the internal calculations of a signal processor, the result of the calculations being rounded. This rounding, too, is a nonlinear operation and care must be taken to use the optimal sequence of the calculations.

3.1 Distribution of Poles and Zeros

The signal processor used (ASP) [3] has the following precision:

12 bits for the coefficients

24 bits for the rounded or truncated results of an arithmetic operation

24 bits for data storage used as delay

40 bits for the results of the accumulator.

The results of the calculations needed for one second-order section must be rounded from 40 to 24 bits. Because the number of bits of the accumulator is higher than that of the rounded results, special attention is paid to deciding which poles and zeros have to be paired within one section. Fig. 2 depicts the effect of a wrong choice for the pairing. It shows the transfer function of a filter that consists of two sections in cascade with individual transfer functions H_1 and H_2. The total transfer function $H_1 \times H_2$ [Fig. 2(a)] is very smooth, but H_1 [Fig. 2(b), solid curve] and H_2 [Fig. 2(b), dashed curve] both have a large quality factor Q. To select a good pole–zero pairing a rather heuristic approach is followed. All the possible combinations to form second-order sections are generated, forming a set S_1. The transfer function of each section is calculated and the combination is rejected when the maximum gain exceeds a certain prescribed level, the remainders forming a set S_2. A new set S_3 is formed containing all the possible combinations of elements out of S_2 necessary to construct the desired filter response. Usually S_3 contains about 10 elements. A final choice is made out of S_3 to form the complete filter. The best choice is the one with the highest signal-to-noise ratio and the least sensitivity to limit cycles.

3.2 Limit Cycles

Limit cycles are fluctuations in the output of a digital filter when the input signal is kept constant. In an audio environment this phenomenon must be avoided. At

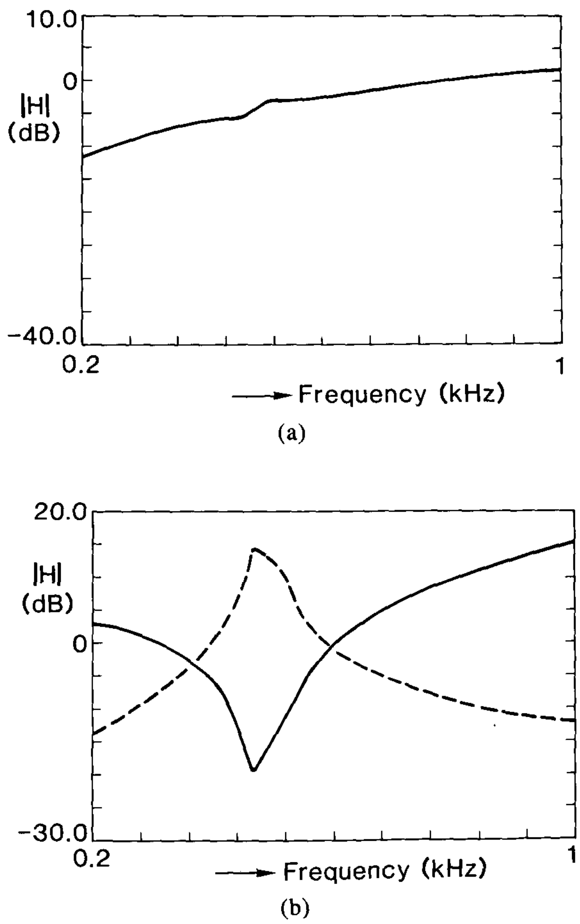

Fig. 2. (a) Amplitude response of two cascaded second-order sections H_1 and H_2. (b) Amplitude responses of second-order sections H_1 (solid curve) and H_2 (dashed curve).

"digital silence" input, limit cycles are particularly inconvenient, and even under normal music conditions care must be taken to avoid limit cycles. Jackson's triangle [4] shows a safe area in which no limit cycles occur. As mentioned in Sec. 3.1, the avoidance of limit cycles is one of the considerations for making the final choice out of set S_3.

3.3 Scaling

When the signal level at the input of the filter is high, overflow may occur at any point in the filter. At places with high Q, appropriate scaling must be applied to avoid overflow. However, the drive level in the filter must be as large as possible to obtain the maximum signal-to-noise ratio.

3.4 A Case Study

As a vehicle we chose a three-way loudspeaker system, equipped with a passive radiator. First the box was measured in an anechoic room to obtain the pressure response of each individual loudspeaker (mounted in the box). The power responses were measured in a reverberation room, and finally the impedances of the loudspeakers were measured. All the data were measured using a digital two-channel signal analyzer. With the aid of the (analog) crossover filter optimization package [1], a number of crossover filter sets were calculated. For different initial values, different loudspeaker polarities, and different weighting values for the target functions, multiple solutions were found. From these solutions, a selection was made for actual listening tests. In this case study we focus on the squawker part of a particular set. The amplitude and the phase of the transfer function of this filter, loaded with the loudspeaker, are shown in Fig. 3 (solid curves). This complex-valued analog transfer function is provided by the crossover filter optimization program and is the target function for the optimization of the z-domain transfer function. This optimization actually minimizes the differences between the analog target function and the z-domain transfer function. The output of the optimization is a z-domain transfer function which fits the desired (analog) transfer function as well as possible. However, the coefficients of the z-domain function are represented in 12 bits in the ASP processor hardware, which is much less accurate than the mainframe computer. The coefficient truncation can have a considerable influence on the transfer function. Depending on different initial values, the number of points in the optimization, and so on, one finds multiple solutions that simulate the same (analog) transfer function. However, all these solutions exhibit large differences for coefficient truncation. To evaluate the influence of the 12-bit coefficient representation or coefficient truncation, we calculate the transfer function with truncated coefficients for two different sets of coefficients. Fig. 4 shows that the influence of the coefficient truncation can be rather dramatic, but the solution (set of z-domain function coefficients) shown in Fig. 3(a) (dashed curve) is hardly affected by the coefficient truncation.

4 CONCLUSIONS

It has been shown that a loudspeaker crossover filter can be simulated by a digital signal processor. This technique can be used for evaluating both passive and active crossover designs. The combination of a numerical optimization of the crossover filters and a digital simulating device is a flexible and powerful design tool, which is being used for the design of new series of commercially available loudspeaker systems.

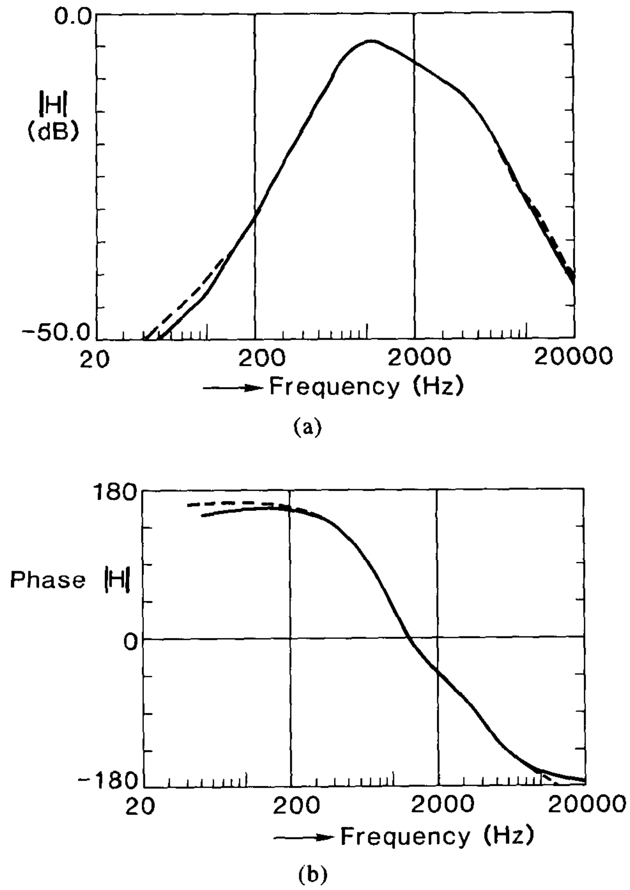

Fig. 3. (a) Amplitude responses. (b) Phase responses. Solid curve—midrange crossover filter loaded with driver (target function); dashed curve—$H(z)$ with truncated coefficients.

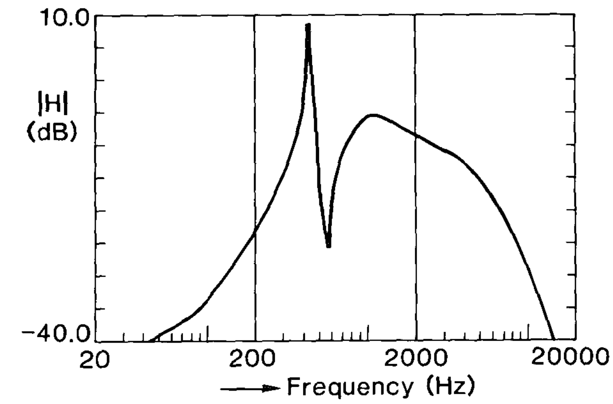

Fig. 4. Amplitude response of transfer function with truncated coefficients.

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APPENDIX
CALCULATION OF AN ANALYTIC FILTER TRANSFER FUNCTION

To calculate an initial estimate for the z-domain transfer function (Sec. 2.1), an analytical s-domain estimate for this transfer function is used. It can be calculated because the topology of the filter and the filter component types are known and the component values are given by the optimization program. The unknown part of the transfer function is an analytical description of the loudspeaker load. This load is estimated by means of a lumped-paramter analogous model of the loudspeaker. However, the calculation of the analytical s-domain transfer function by hand is quite cumbersome and prone to error. Therefore a symbolic computation [9] program REDUCE [10] is used to calculate the transfer function in analytical form. This package also generates a FORTRAN or Pascal program source, which calculates the poles and zeros of the transfer function numerically. To this end the filter and impedance loads are modeled in a ladder network [11], that is, a cascade of two-port sections. Each section is represented by a matrix with symbolic elements, and the total transfer function is obtained from the product of the individual matrices.

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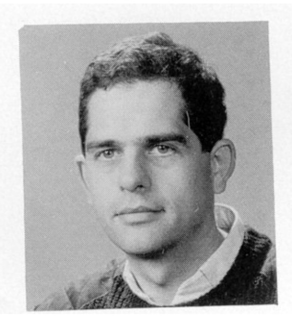

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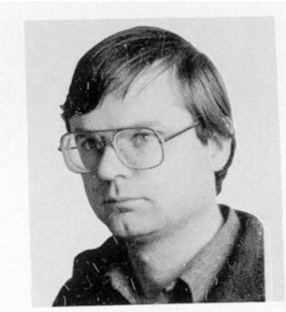

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Mr. Aarts has published a number of technical papers and reports and holds several patents in his field. He is a member of the AES.

Arie J. M. Kaizer was born in Amsterdam, The Netherlands, in 1950. He received a B.Sc. degree in electrical engineering in 1973 and an M.Sc. degree in electrical engineering in 1976, both from the Twente University of Technology, Enschede, The Netherlands. In 1986 January he received a Ph.D. degree from the Eindhoven University of Technology. His thesis was titled "On the Design of Broadband Electrodynamic Loudspeakers and Multiway Loudspeaker Systems." He joined the acoustics group of Philips Research Laboratories in 1976 and has since worked in the field of electroacoustics. In 1985 he joined the acoustics group

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Dr. Kaizer has published and presented a number of technical papers and reports, and he also holds several patents in this field. In 1985 he received (together with C. P. Janse) the Audio Engineering Society Publication Award for the paper "Time-Frequency Distributions of Loudspeakers: The Application of the Wigner Distribution."