A New Method for the Design of Crossover Filters*

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A new method is presented for the design and evaluation of loudspeaker crossover filters. The desired system characteristic can be prescribed by a (complex) acoustic transfer function rather than an electrical one only. It may be derived from conventional filters or based on a measured filter from a reference (favorable) system. Double blind listening tests are performed to verify subjectively the similarity between the reference system and its experimental counterpart. The drivers of the experimental loudspeaker are preceded by digital filters, enabling the imitation of several different favorable loudspeakers. Multidimensional scaling techniques are applied to represent the results of the listening tests. These results affirm the strength of the design method.

0 INTRODUCTION

The crossover or frequency-dividing network plays an important role in the performance of high-quality loudspeaker systems. Individual drivers do not have the capability of reproducing all frequencies in the entire audio range with sufficient directional properties. The crossover network allows various drivers, each suited to a particular frequency range, to be combined into a system covering a wide frequency range. Because of its importance, since the early paper by Hilliard and Kimball [1], a great deal has been published on this subject during the past 50 years. See [2], [3] for an overview of the many designs available.

The design of crossover networks can be divided into three approaches.

1) The first approach is to use one of the well-known filter families as crossover filter, such as the Butterworth types or the celebrated Linkwitz–Riley [4], [5] filters. Other possible arrangements are the delay-derived high-slope filters [6], the constant-voltage filters [7], all-pass systems [8], magnitude complementary filters [9], and the filler–driver design [10], [11]. In contrast to the other filters, the filler–driver is more than a filter only. It uses an additional driver.

However, most analyses are concerned with the electrical part of the filter, thereby ignoring the acoustic behavior of the drivers and the cabinet. Usually the drivers are considered to be ideal; the peaks and dips in the operating frequency range are ignored, as is the frequency dependence of the electric input impedance. The design of a crossover network can be a difficult and tedious affair when these imperfections are taken into account.

2) The second approach is by numerical optimization [12]–[16]. These programs have proved their usefulness in the design of crossover networks when the imperfections mentioned are taken into account. Filters known from network theory or existing crossover designs are used as a first approximation or seed value for the numerical algorithm. During optimization the filter component values are determined in such a way that the difference between the desired and the calculated acoustic response of the loudspeaker system is minimized.

3) The third approach, which is the subject of this paper, uses a real existing or a hypothetical (favorable) system as a reference to supply a target function for the system under design. Each driver plus crossover filter of that reference system acts individually as an electroacoustic target function for the crossover filter to be optimized. The optimization process yields a set of coefficients for the digital filters preceding each driver of the experimental loudspeaker. By only changing the coefficients of these digital filters, it is possible to let one and the same experimental loudspeaker sound very similar to different reference systems.

In the following we discuss the design of these digital

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filters and present examples with two different reference systems. The similarity between the reference system and its experimental counterpart is verified by both objective and subjective means. The latter is performed by listening tests using triadic comparisons. The results are represented using multidimensional scaling techniques.

1 AIM

In a previous paper [17] a tool was described for the simulation of real existing or hypothetical crossover filters. It consists of a number of computer programs and a digital signal processor. A set of filters was calculated by a computer program [12] that performs the (nonlinear) optimization of several functions. These functions are usually the sound pressure response and the sound power response. Listening experiments in which the judges could select from these filters underscored again the powerful influence of crossover filters. With one and the same system, that is, the cabinet and three drivers, very different acoustical images could be effected. After listening tests a final choice was made for the construction of the analog filter. Another possible approach is to use a real existing loudspeaker set as an object function.

The aim is to make the experimental system response equal to that of the reference system. A simple method would be to place an equalizer before the experimental system and to adjust it in such a way that both the reference and the experimental systems will give the same response at a certain field point. One step further is to equalize each driver instead of the entire system. One could consider this method as a combination of an equalizer and a crossover filter. In the new case each driver has its "own" equalizer, thereby giving the possibility of varying the directional behavior with only a minimal effect on the direct or on-axis sound pressure response. In order to have the same acoustical image for both the reference and the experimental system, the crossovers of the experimental box have to be chosen in a such a way that the total transfer function (acoustic and electric) of each driver is the same as that of the corresponding reference drivers.

Let the acoustical reference transfer function of the jth driver be defined by

$$H_j^{\mathrm{a}}(s, \boldsymbol{x}) = \frac{p_j(s, \boldsymbol{x})}{V_j^{\mathrm{t}}(s)} \tag{1}$$

where p_j is the sound pressure level at position $\boldsymbol{x}$, V_j^{t} is the voltage across the terminals of the jth driver, and s is the complex frequency variable. The electrical transfer function of the jth filter is defined by

$$H_j^{\mathrm{e}}(s) = \frac{V_j^{\mathrm{t}}(s)}{V_{\mathrm{in}}(s)} \tag{2}$$

where V_{in} is the input voltage of the entire system. Then the total driver response for the jth driver yields

$$H_j^{\mathrm{t}}(s, \boldsymbol{x}) = H_j^{\mathrm{a}}(s)\, H_j^{\mathrm{e}}(s) \tag{3}$$

and the total system response for n drivers yields

$$H_{\mathrm{t}}(s, \boldsymbol{x}) = \sum_{j=1}^{n} H_j^{\mathrm{a}}(s)\, H_j^{\mathrm{e}}(s)\ . \tag{4}$$

The intention is to make the experimental system response $\hat{H}_{\mathrm{t}}(s, \boldsymbol{x})$ equal to that of the reference system. This can be accomplished if the drivers of the experimental system are preceded by a filter having

$$\hat{H}_j^{\mathrm{e}}(s, \boldsymbol{x}) = \frac{H_j^{\mathrm{t}}(s, \boldsymbol{x})}{\hat{H}_j^{\mathrm{a}}(s, \boldsymbol{x})} \tag{5}$$

as transfer function. As Eq. (5) shows, $\hat{H}_j^{\mathrm{e}}(s)$ depends on $\boldsymbol{x}$. To cancel this dependence we require the physical position of the drivers on the baffle and the size of the drivers and the cabinet to be comparable for the reference system and the experimental counterpart. If this requirement is satisfied, we assume that the polar responses (radiational behavior) of both systems have been made similar when the responses at one particular position are equal.

The filter $\hat{H}_j^{\mathrm{e}}(s, \boldsymbol{x})$ is approximated for a chosen $\boldsymbol{x} = \boldsymbol{x}_0$ by a digital filter $H_j^{\mathrm{d}}(s)$. The coefficients $\boldsymbol{c}$ of that filter have to be calculated by means of an optimization procedure, that is, by minimizing the functional

$$\varepsilon_j'(\boldsymbol{c}) = \int_{\omega_{\mathrm{b}}}^{\omega_{\mathrm{e}}} [|\hat{H}_j^{\mathrm{e}}(\mathrm{i}\omega, \boldsymbol{x} = \boldsymbol{x}_0) - H_j^{\mathrm{d}}(\mathrm{i}\omega)|]^2\, \mathrm{d}\omega \tag{6a}$$

where ω_{b} and ω_{e} are the beginning and ending points of the frequency band of interest. In practice, however, the reference transfer functions are obtained by measurement and are therefore known at r discrete frequencies only. This replaces the functional in a form

$$\varepsilon_j(\boldsymbol{c}) = \sum_{k=1}^{r} W_k\, [|\hat{H}_j^{\mathrm{e}}(\mathrm{i}\omega_k, \boldsymbol{x}_0) - H_j^{\mathrm{d}}(\mathrm{i}\omega_k)|]^2 \tag{6b}$$

where W_k are (frequency-dependent) weighting factors. The coefficients of the digital filter are calculated by means of an optimization procedure based on a modified Gauss–Newton method [18]. Because the variables in Eq. (6) are complex, both amplitude and phase are taken into account.

2 DESIGN EXAMPLES

In order to test the practical outcome of the design method discussed, two different brands of three-way loudspeaker systems were chosen as a vehicle. These systems are comparable with each other with respect to cabinet volume, driver size, and commercial price. However, their sound was very different. This is due to a different on-axis (anechoic chamber) response as well as a different polar response. Due to the latter, the final perceived sound depends on the listening environment. First we consider the anechoic chamber response and then a normal listening environment response.

2.1 Anechoic Chamber Response

For each driver of both systems, labeled brand A and B, respectively, the sound pressure responses were measured at a distance of 3 m right in front of a point between tweeter and squawker. The total system responses were determined too and are shown in Fig. 1(a) and (b) as solid curve for systems A and B, respectively. The optimization procedure as discussed in the previous section yields a set of coefficients A_a if system A is used as a target and B_a if system B is used. The subscript a denotes the anechoic chamber environment. With the experimental setup depicted in Fig. 2, the transfer functions were calculated for the digital filters loaded with either set A_a or B_a. The results are plotted in Fig. 3. Fig. 3(a) gives the responses of the filters for the woofer, squawker, and tweeter using set A_a, and Fig. 3(b) gives the responses using set B_a. As these figures show, the SPL responses are quite different from the responses known from the classic designs.

Each filter is placed before the appropriate driver of the experimental system. The sound pressure of the total system as well as that of the individual drivers were measured under the same conditions as those of the target systems. The responses are drawn in Fig. 1(a) and (b) as dashed curves using sets A_a and B_a, respectively. It should be emphasized that both (dashed-curve) responses were measured from one and the same experimental loudspeaker system with only the coefficients of the filters being changed. As the curves show, the responses of target and experimental systems are

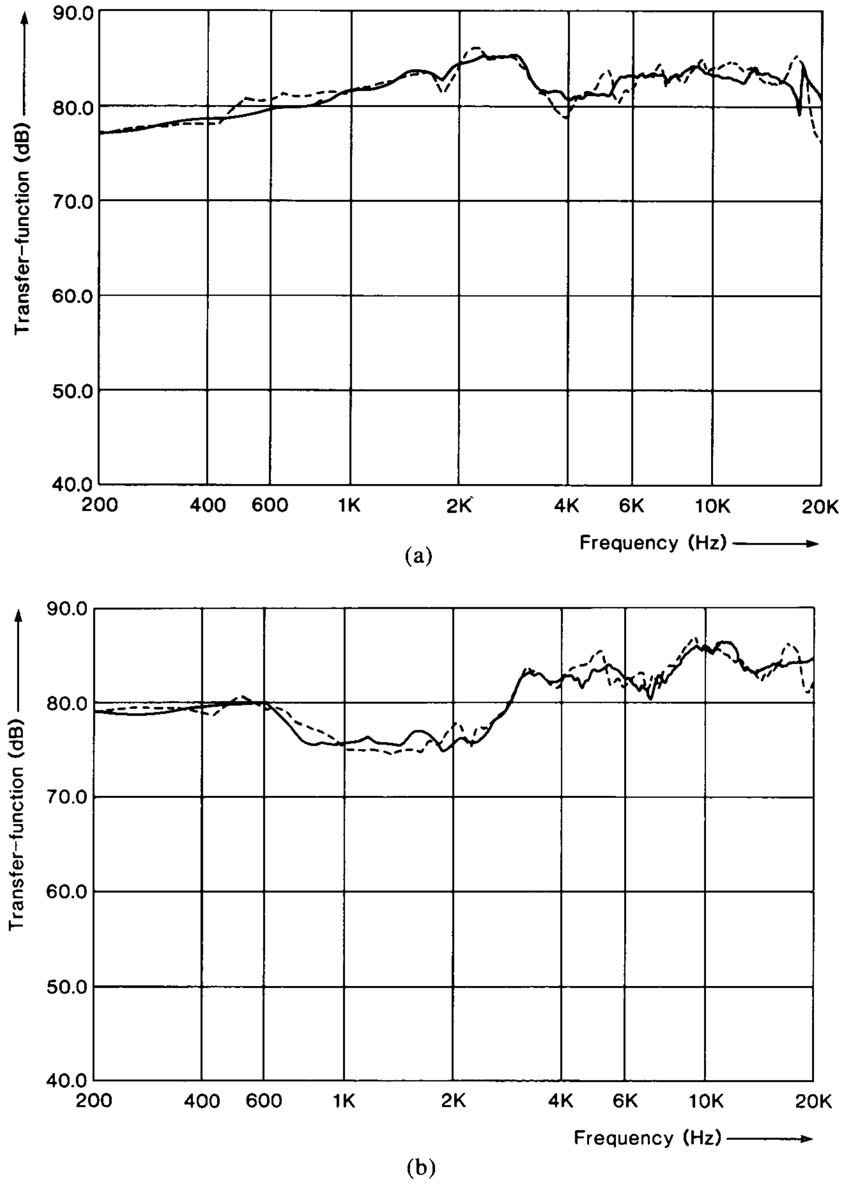

Fig. 1. SPL of (a) system A and (b) system B (solid) and experimental system (dashed).

very similar. But even more important, this also holds for the corresponding individual drivers, for both magnitude and phase. The phase equality is crucial because the summation in Eq. (4) can be very sensitive to phase errors.

Although the curves in Fig. 1 show good agreement, subjective comparisons were also made, and these are discussed in Sec. 3.

2.2 Listening Room Response

As pointed out by several others, the total acoustic power of a loudspeaker system is very important, especially in relation to crossover designs [19], [20]. Systems with the same on-axis anechoic response can sound quite different in a living room. Staffeldt [21] noticed the resemblance between the average living room response and the total acoustic power response measured in a reverberation room, for example.

Because of the important influence on the response exerted by the listening environment, the response of system A was determined in a listening room ($V = 210$ m^3 and, at 500 Hz, $T_{60} = 0.4$ s). Target system A was placed on the ground. With a microphone at 3-m distance and 1-m height, the responses of the individual drivers (preceded by their filters) and that of the total system were measured. Thereafter the responses of the experimental system were measured under the same conditions. Following the same procedure as described in the previous section, a set of coefficients A_l was determined. The subscript l denotes the listening room as measuring environment. Due to the acoustic modes in the listening room, the measured transfer functions are rather hectic. To compare the target and the experimental loudspeaker responses, some spatial averaging is applied. The same microphone used to measure the target functions was placed on a rotating boom. The orbit of the microphone described a circle of 250-mm diameter, parallel to the floor at 1-m height. The spectra obtained in this manner are shown in Fig. 4. The solid curve is the measured response of target system A, while the dot–dash curve represents the response for the experimental system using filters with coefficient set A_l. As the figures show, there is a good similarity between both systems. The corresponding driver responses display good agreement also. This is shown in Fig. 5 for the squawker response of target A (solid curve) and the experimental one (dotted curve). As noted, to affirm the similarity between both systems listening tests were carried out, and these are discussed in the following.

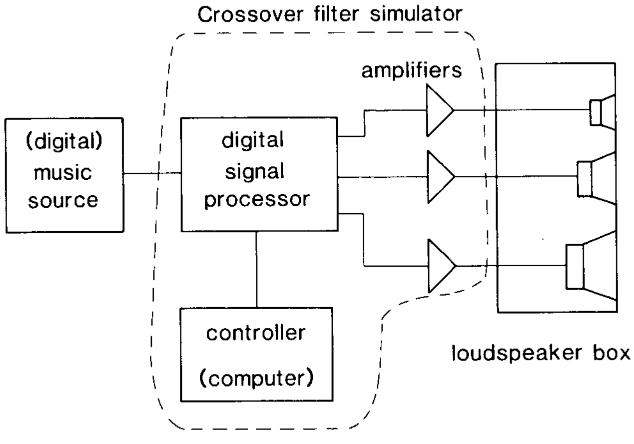

Fig. 2. Experimental setup.

3 SUBJECTIVE EVALUATION

At present the subjective evaluation of loudspeaker systems is still a crucial part of the design process. No good criteria are yet available for translating objective measurements into a prediction of the perceived sound. Although several others [21]–[27] have tried this and achieved some success, listening remains important and is fortunately not the most boring part of the design procedure.

The aim was to make the experimental system similar to several other reference systems by only changing certain filter parameters. An obvious method to verify

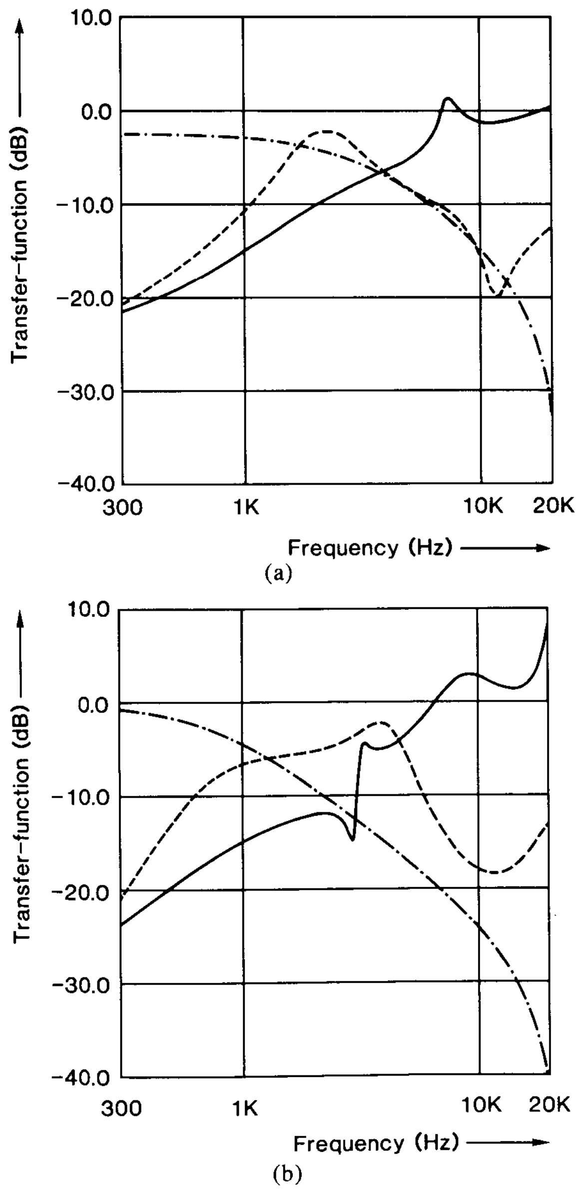

Fig. 3. Filter responses (a) using set A_a. (b) Using set B_a.

the result would be to place the reference and experimental systems next to each other, and subsequently to listen to them to discern whether they are indistinguishable or, if they are not, by how much they differ. However, an insuperable problem arises, as follows. If one switches between the two systems, it is easy to identify both systems because of the readily perceived change in the direction of the sound. A fair comparison would only be possible if the systems to be compared were placed in the same physical position. This makes a fast comparison, which is considered to be very important, impossible. We agree with Shanefield's opinion [26]: "Fast comparisons provide sensitivity to real differences, and long-term listening simply encourages imaginary ones. But both should be tried." Yet fast comparisons are possible by listening not directly to the systems but to recordings of them, as discussed in the next sections.

3.1 First Experiment

Four (mono) music pieces, with a duration of 2 min each as well as some test signals were recorded successively on track 1 of an 8-track tape recorder. On the other tracks recordings of the various systems described in Table 1 were made in an anechoic chamber. One loudspeaker system at a time was present in the anechoic chamber, so that each system occupied the same place during the recordings. The microphone and cabinet positions for the systems under test were identical to those described in Sec. 2.1.

To have some additional reference material, recordings were made of a third brand-name reference loudspeaker labeled C, and of the experimental loudspeaker equipped with filters using coefficient set D_a. The latter was intended to imitate a hypothetical ideal system, having a flat pressure response and a gradually decreasing (acoustic) power response. The first track of the tape recorder was used as the source for the other six recordings. At each recording session the tape was rewound, and because the recorder has a combined playback and record head, each track can be played back synchronously with the others. Finally, it was then possible to listen to each system and switch instantaneously between them. The tape can be played back in any environment using high-quality loudspeaker systems or headphones.

3.1.1 Aim

This experiment used similarity judgments between music pieces reproduced by different reference loudspeaker systems and their experimental counterparts. Multidimensional scaling was used to examine how

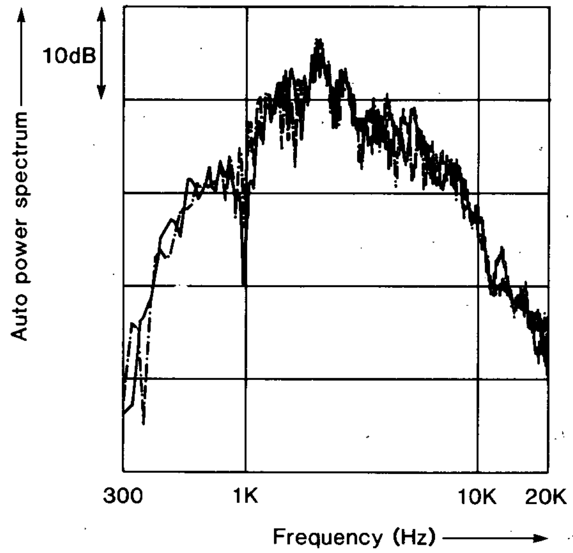

Fig. 5. Squawker SPL of system A (solid) and experimental system (dot–dashed).

Table 1. Track allocation for first experiment.

Track	
1	Direct recording from Compact Disc player
2	Experimental system using coefficient set D_a
3	System C
4	Experimental system using coefficient set A_a
5	System A
6	Experimental system using coefficient set B_a
7	System B

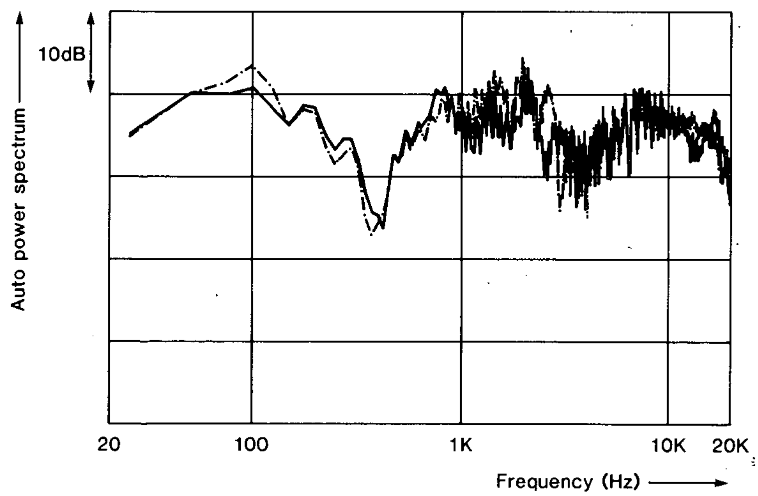

Fig. 4. SPL of system A (solid) and experimental system (dot–dashed).

many subjective dimensions are needed to describe the perceptual space, and to scale the measured perceptual differences.

3.1.2 Method

The method of triadic comparisons (see Appendix) was used to obtain perceptual distances between the stimuli. One can form 35 different triads from seven stimuli. Presentation of all triads would have loaded the subjects too much. Therefore a subset of 14 triads was taken, which still presented all loudspeaker systems to the subjects an equal number of times. Such a subset is an incomplete balanced block design [37], [38]. Twelve subjects participated in the experiment. All stimuli were presented by headphones under diotic (both ears the same signal) conditions. Using the experimental setup as depicted in Fig. 6, a triad was selected and remained fixed as long as the subject wished. The triad components were always labeled 1, 2, and 3. The subjects did not know the relationship between these numbers and the real channel numbers of the tape recorder. The subjects could freely select one out of the three and switch between them as much as they wanted. After answers were given to the questions "Which are the most similar?" and "Which are the least similar pairs of the three?" the test was stopped and a new triad was chosen. This was repeated until all 14 triads had been considered. In the meanwhile the tape recorder played the music pieces sequentially and then rewound automatically. The stimuli were presented through high-quality headphones. The subjects could adjust the playback level to their own preference, but they were allowed to do so only once. A session of 14 triads took approximately 40 min.

3.1.3 Results

The experiment resulted in a triangular similarity matrix for each subject. In order to generate a more graded matrix, the similarity indices of all the subjects were gathered and normalized, the normalization factor being the total number of occurrences of that pair. So the maximum of an entry is 2. The results are listed in Table 2. Consequently, as Table 2 shows, there was a consensus between all subjects for the pair 6, 7.

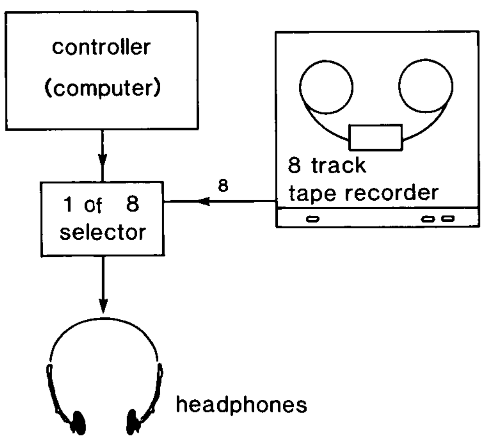

Fig. 6. Experimental setup for listening tests.

The multidimensional scaling program KYST2a was used to scale these data (see Appendix). The program was used in a nonmetrical mode, using a normal Euclidian space (Minkowski parameter = 2). The program calculated, with Stress 1 = 0.008 in two dimensions, a configuration as depicted in Fig. 7. The Stress 1 values for one and three dimensions were 0.231 and 0.01, respectively. These values indicate that a two-dimensional representation describes the perceptual space very well. The labels in the figure correspond to the channel numbers of the tape recorder as given in Table 1. From Fig. 7 three clusters can be distinguished. The similarity between system A and its experimental counterpart, and similarly of system B and the experimental system, is obvious. The third cluster led to the decision to denote the experimental system using coefficient set D_a as a true high-fidelity system: it is very similar to the original Compact Disc music.

3.2 Second Experiment

In the first experiment only one microphone position in an anechoic chamber was used to record the music on tape. In order to examine the influenc of a deviation of the microphone position with respect to the position used to calculate the filters, some other recordings were made with the microphone displaced. According to the theory of Sec. 1, only a minor influence should be observed. Another difference compared with the first experiment concerns the recording room; this time a listening environment was used instead of an anechoic chamber.

One loudspeaker system at a time was present in the listening room, so that each system occupied the same

Table 2. Normalized similarity half-matrix for all subjects.

1	—						
2	1.6	—					
3	1.9	1.6	—				
4	0.4	1.2	1.1	—			
5	1.2	1.1	0.8	1.9	—		
6	0.4	1.1	0.1	0.9	0.9	—	
7	0.4	1.0	0.6	0.3	0.6	2.0	—
	1	2	3	4	5	6	7

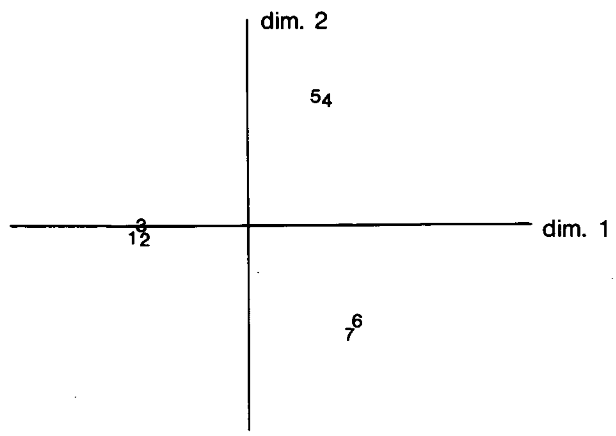

Fig. 7. Perceptual configuration. See Table 1 for legend.

place during the recordings. The others were out of the room to avoid their disturbing the sound field. The cabinet position for the systems under test was identical to that described in Sec. 2.2, but the microphone position was changed to positions M_2 and M_3. Position M_2 was right in front of the system at a distance of 4 m, M_3 was at the same distance, but with a deviation of 25°.

As described in Sec. 3.1, track 1 of the tape recorder was used as the source for the recordings made according to Table 3. This experiment enabled the subject, while listening to a system in a particular room, to move from a chair at position M_2 to one at position M_3 instantaneously without breaking arms or legs, which probably would have been the result of such a jump.

As in the first experiment, triadic comparisons were used. Track 1 of the tape was not included this time. This means that there were four possible triads. In order to increase the number of triads, each triad was repeated three times, each time with a different order within the triad, yielding a total of 12 triads. Fifteen subjects participated in this experiment. They used on average 35 min per session.

3.2.1 Results

The similarity matrices obtained from the subjects were pooled to form one matrix and normalized as in the previous experiment. The result is shown in Table 4.

Again the MDS program was used in a nonmetrical mode, using a normal Euclidian space to calculate a configuration in two dimensions out of this matrix. The result is depicted in Fig. 8. The labels in the figure correspond with the channel numbers of the tape recorder, as shown in Table 3. The similarity between system A and the experimental counterpart, both at position M_2, is obvious; the same holds for both systems at position M_3. The perceptual distance for changing the microphone position while using the same system is larger than the distance for changing to the other system while keeping the microphone position fixed. So we conclude that a deviation of the listening (microphone) position does not influence the similarity.

Table 3. Track allocation for second experiment.

Track	
1	Direct recording from Compact Disc player
2	System A, position M_2
3	Experimental system coefficient set A_1, position M_2
4	System A, position M_3
5	Experimental system coefficient set A_1, position M_3

Table 4. Normalized similarity half-matrix for all subjects.

2	—			
3	1.9	—		
4	0.7	0.4	—	
5	0.6	0.5	1.8	—
	2	3	4	5

4 CONCLUSIONS

A new technique is presented for the design of crossover filters. It is shown to be a powerful and versatile one. With these filters preceding (nonideal) drivers, the acoustical (on-axis and polar) responses can be made similar to prescribed ones. The prescribed responses can be obtained from a hypothetical system or from the measurement of an existing one. The similarity holds not only for the total system, but also for the individual drivers.

Listening tests, using triadic comparisons, were used to affirm the claimed similarity. The influence of extraneous factors was excluded as far as possible by using a special listening arrangement. Thus the subjects were not listening directly to the loudspeaker systems, but to tape recordings of them. Multidimensional scaling, shown to be a very useful technique, was applied to represent the results of the listening tests.

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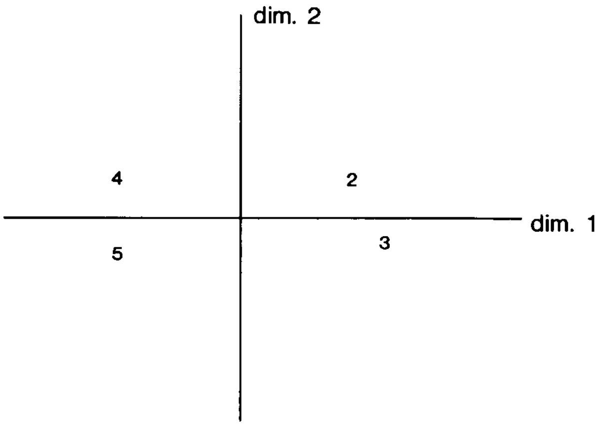

Fig. 8. Perceptual configuration. See Table 3 for legend.

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APPENDIX
MULTIDIMENSIONAL SCALING

A.0 Introduction

A problem encountered in many disciplines is how to measure and interpret the relationships between objects. A second problem is the lack, in general, of a mathematical relationship between the perceived response and the actual physical measure. Sometimes the relations are rather vague. How much does the character of one person look like that of another? Or, concerning this paper, how much are different loudspeakers alike? How do we measure and what scale do we need? In the following we discuss some scales and techniques and give an example.

A.1 Scaling

The purpose of scaling is to quantify qualitative data. Scaling procedures attempt to do this by using rules that assign numbers to qualities of things or events. Multidimensional scaling (MDS) is an extension of univariate scaling. It is simply a useful mathematical tool that enables us to represent the similarities of objects spatially as in a map.

MDS models may be either metric or nonmetric, depending on the scale of measurement used in collecting the data. For metric scaling the collected data should be at interval or ratio scale. In the former case the unit of the "yardstick," used for measuring the phenomenon, as well as the zero point (offset), are unknown. For a ratio scale the zero point is known, but there is an unknown scaling factor. For nonmetric MDS only the ranking order of the data is used; the data are or are used at ordinal level. However, it is sometimes possible to recover metric distances, obtained by nonmetric MDS, as will be shown in an example.

In order to obtain a spatial map from an MDS computer program, we only need to apply a set of numbers as input. To all (or most) combinations of pairs out of a group of objects, a number is assigned which expresses the similarity between the objects of that pair. Such numbers are sometimes referred to as proximities. MDS procedures will then represent objects judged similar to one another as points close to each other in the resultant spatial map. Objects judged to be dissimilar are represented as points distant from each other.

MDS programs which use direct similarity measures as input have the advantage of being low in experimental contamination. They do not require a priori knowledge of the attributes of the stimuli to be scaled.

A.2 Example

An obvious procedure for obtaining similarity data is to ask people directly to judge the "psychological distance" of the stimulus objects. Another way is the method of triadic comparisons [28], [29]. This has the advantage that it simplifies the subject's task, because only the ranking order of three presented stimuli is asked for. However, there can be some drawbacks, as pointed out in Roskam [30]. A practical problem arises when for a complete experiment the number of triads (all possible combinations of three stimuli out of the set of all stimuli) is considered too large. It can be reduced by using an incomplete balanced block design (IBBD) [37], [38].

An example of MDS using triadic comparisons follows. Suppose someone with a good topographical knowledge of The Netherlands is asked to give the nearest and the most distant two cities out of three given cities. The same question is asked for three other cities, and so on. Thus each distance from one city to another (each out of a total list of, for example, 14 cities) is considered. A matrix M can be constructed so that for the three cities (i, j, k) the two closest together, such as (i, j), contribute 0 points to the matrix element $M(i, j)$; the next closest pair, such as (j, k), add 1 point; and the remaining pair add two points to $M(i, k)$. The (dissimilarity) matrix obtained in this way resembles an ordinary distance table. If the phrases most distant and nearest in the question are interchanged, one obtains a similarity (data) matrix.

Instead of relying on a topographer we used an ordinary distance table as input for the program. The program we applied was KYST-2A, pronounced "kissed," formed from the initials Kruskal, Young, Sheppard, and Torgerson [31]. It gives the coordinates of the cities in one or more dimensions. The analysis was carried out for both the metric case (with linear regression) and the nonmetric case. In the latter, the actual number of miles was not used. But the ranking order of the calculated interpoint distances should be, as far as possible, the same as the ranking order of the interpoint distances in the given distance matrix. The results of both the metric and the nonmetric cases were practically the same. Only the results of the latter are discussed in the following.

All calculations were carried out in the Euclidian space (Minkowski's parameter = 2). A measure of the goodness of fit between both rankings is called stress, which can in some degree be compared with a least-squares sum in an ordinary fitting procedure. The stress value in this particular case is 0.249 for one dimension, 0.028 for two dimensions, and 0.013 for three dimensions. It appears that a two-dimensional fit is a good one. The decrease of stress in three dimensions is rather weak, while the deterioration due to a low dimensionality is obvious. The results of the calculations are plotted in Fig. 9. The solid points are the real locations, whereas the small circles represent the calculated places. As the figure shows, in this particular case it is possible to derive metric data from nonmetric input data. The orientation of the map is arbitrary; there is no real north–south axis. For convenience only, the contour of The Netherlands and a compass needle are drawn.

A.3 Some Bibliographic Notes

A short but authoritative introduction to MDS is Kruskal and Wish [32]. A comprehensive survey of the development of MDS is that of Carroll and Arabie [33], which cites 334 references, mostly published during the 1970s. More recent are the surveys by Young [34] and (on general scaling) by Gescheider [35]. The latter is more about sensory and cognitive factors that affect psychophysical behavior rather than about measurement and computational aspects. A review intended for a wide general scientific audience, concerning the models and applications of MDS and cluster analysis, has been given in Shepard [36].

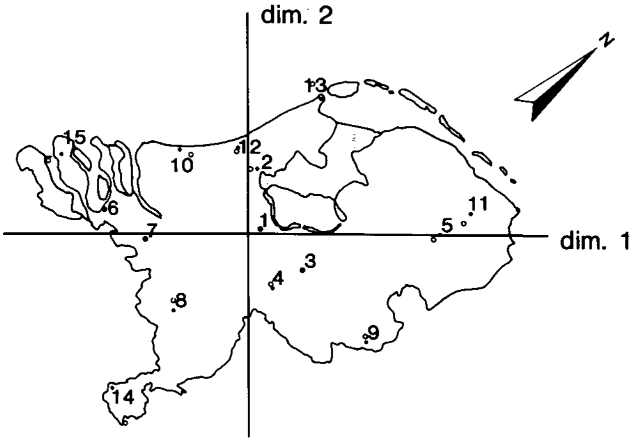
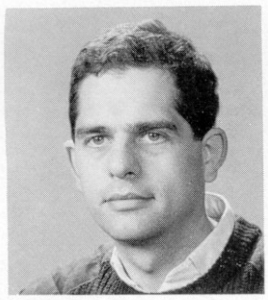


Fig. 9. Configuration with real (solid) and calculated (circles) locations.

THE AUTHOR



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